# CP asymmetries in scalar bottom quark decays 

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Abstract: We propose CP asymmetries based on triple product correlations in the decays $\tilde{b}_{m} \rightarrow t \tilde{\chi}_{j}^{-}$with subsequent decays of $t$ and $\tilde{\chi}_{j}^{-}$. For the subsequent $\tilde{\chi}_{j}^{-}$decay into a leptonic final state $\ell^{-} \bar{\nu} \tilde{\chi}_{1}^{0}$ we consider the three possible decay chains $\tilde{\chi}_{j}^{-} \rightarrow \ell^{-} \overline{\tilde{\nu}} \rightarrow \ell^{-} \tilde{\nu}_{1}^{0}, \tilde{\chi}_{j}^{-} \rightarrow$ $\tilde{\ell}_{n}^{-} \bar{\nu} \rightarrow \ell^{-} \bar{\nu} \tilde{\chi}_{1}^{0}$ and $\tilde{\chi}_{j}^{-} \rightarrow W^{-} \tilde{\chi}_{1}^{0} \rightarrow \ell^{-} \tilde{\nu} \tilde{\chi}_{1}^{0}$. We consider two classes of CP asymmetries. In the first class it must be possible to distinguish between different leptonic $\tilde{\chi}_{j}^{-}$decay chains, whereas in the second class this is not necessary. We consider also the 2 -body decay $\tilde{\chi}_{j}^{-} \rightarrow W^{-} \tilde{\chi}_{1}^{0}$, and we assume that the momentum of the $W$ boson can be measured. Our framework is the minimal supersymmetric standard model with complex parameters. The proposed CP asymmetries are non-vanishing due to non-zero phases for the parameters $\mu$ and/or $A_{b}$. We present numerical results and estimate the observability of these CP asymmetries.

Keywords: Supersymmetry Phenomenology, CP violation.

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## 1. Introduction

In the Minimal Supersymmetric Standard Model (MSSM) [1], 2] the higgsino mass parameter $\mu$ and several of the Supersymmetry (SUSY) breaking parameters are complex in general. Among the SUSY breaking parameters the trilinear scalar couplings $A_{f}$ and two of the gaugino mass parameters $M_{1}$ and $M_{3}$ ( $M_{2}$ is usually chosen to be real by redefining the fields) can be complex.

Current experimental upper bounds on the electric dipole moments (EDM) impose restrictions on the SUSY parameters that appear in supersymmetric models, in particular on their phases. To which extent the size of the phases have to be restricted, however, strongly depends on the underlying model. For instance, while only relatively small values of the phase of $\mu,\left|\phi_{\mu}\right| \lesssim 0.1$, are allowed in several versions of the MSSM with selectron masses of the order 100 GeV [3], this restriction may disappear if lepton flavour violating terms are included (4) or if the masses of the first and second generation scalar fermions are large (above the TeV scale) while the masses of the third generation scalar fermions are small (below 1 TeV ) 5]. Recently it has been pointed out that for large trilinear scalar couplings $|A|$ one can simultaneously fulfill the EDM constraints of electron, neutron, and that of the atoms ${ }^{199} \mathrm{Hg}$ and ${ }^{205} \mathrm{Tl}$, where at the same time, $\phi_{\mu} \sim O(1)$ [6]. The restictions
on the size of the phases of the trilinear couplings of the 3rd generation scalar fermions are far less important as their contributions to the EDMs appear only at two-loop level [7].

The various CP phases can have a big influence on the production and decay of supersymmetric particles. In particular the influence of the phases $\phi_{A_{\tau, t, b}}$ of the trilinear scalar coupling parameters on various observables (e.g. scalar fermion masses, cross sections, decay widths) can be important [8, 9]. However, a measurement of solely CP-even observables cannot be sufficient to unambiguously determine the SUSY parameters. Moreover, in order to clearly demonstrate that CP is violated, CP-odd observables have to be measured. Rate asymmetries have been proposed where the influence of the SUSY CP phases arise due to loop corrections (see for instance 10]). Another important class of CP-odd observables are based on triple product correlations (for an introduction see [11]). They arise already at tree-level and allow to define various CP asymmetries which are sensitive to the different CP phases. Such CP asymmetries have been proposed and analyzed for various SUSY processes (see for instance [12, 13]).

Recently, it has been shown [13] that triple product correlations among the decay products of the scalar top decay $\tilde{t} \rightarrow t \tilde{\chi}^{0}$ followed by the decays of $t$ and $\tilde{\chi}^{0}$, allow us to obtain information on CP violation in the scalar top system. Along the same line of the study performed in [13], in the present paper we analyze triple product correlations that arise in the decays of the scalar bottoms $\tilde{b}_{m}$. We focus on the influence of CP violation in the scalar bottom system, in particular on the influence of the phase of the trilinear scalar coupling parameter $A_{b}, \phi_{A_{b}}$.

We study the decay

$$
\begin{equation*}
\tilde{b}_{m} \rightarrow t \tilde{\chi}_{j}^{-}, \tag{1.1}
\end{equation*}
$$

followed by the subsequent decays of the top quark $t$ and the chargino $\tilde{\chi}_{j}^{-}$. We work in the approximation where $t$ and $\tilde{\chi}_{j}^{-}$are both produced on mass-shell. As the top quark does not form a bound state this implies that both $t$ and $\tilde{\chi}_{j}^{-}$decay with definite momenta and polarizations. Their polarizations can be retrieved from the distributions of their decay products. We consider the decays of the top quark

$$
\begin{equation*}
t \rightarrow b W^{+} \quad \text { and } \quad t \rightarrow b l^{+} \nu_{l}(b c \bar{s}), \tag{1.2}
\end{equation*}
$$

and the following three possible decay chains for $\tilde{\chi}_{j}^{-}$:

$$
\begin{align*}
& \tilde{\chi}_{j}^{-} \rightarrow \ell_{1}^{-} \overline{\tilde{\nu}} \quad \rightarrow \ell_{1}^{-} \tilde{\chi}_{1}^{0},  \tag{1.3}\\
& \tilde{\chi}_{j}^{-} \rightarrow \tilde{\ell}_{n}^{-} \bar{\nu} \rightarrow \ell_{2}^{-} \tilde{\nu}_{1}^{0},  \tag{1.4}\\
& \tilde{\chi}_{j}^{-} \rightarrow W^{-} \tilde{\chi}_{1}^{0} \rightarrow \ell_{3}^{-} \bar{\nu} \tilde{\chi}_{1}^{0}, \tag{1.5}
\end{align*}
$$

which lead to the final states

$$
\begin{equation*}
\tilde{\chi}_{j}^{-} \rightarrow \ell^{-} \bar{\nu} \tilde{\chi}_{1}^{0}, \quad \ell=e, \mu, \tau . \tag{1.6}
\end{equation*}
$$

We shall consider each of the decays (1.3), (1.4), (1.5) separately. The subscript of the leptons, $\ell_{1}, \ell_{2}, \ell_{3}$, is used in order to distinguish them in the different decay chains. For
simplicity we shall work in the narrow width approximation for the intermediate particles in (1.3)-(1.5), i.e. we assume that these particles are produced on-mass-shell.

We consider also the 2-body decay of $\tilde{\chi}_{j}^{-}$:

$$
\begin{equation*}
\tilde{\chi}_{j}^{-} \rightarrow W^{-} \tilde{\chi}_{1}^{0} \tag{1.7}
\end{equation*}
$$

assuming the momentum of the final $W$ boson can be reconstructed, which is possible for hadronic decays.

We consider the triple products

$$
\begin{equation*}
\mathcal{O}=\mathbf{q}_{1} \cdot\left(\mathbf{q}_{2} \times \mathbf{q}_{3}\right) \equiv\left(\mathbf{q}_{1} \mathbf{q}_{2} \mathbf{q}_{3}\right), \tag{1.8}
\end{equation*}
$$

where $\mathbf{q}_{i}$ are any 3-vectors of the particles in the considered process. With the help of the triple products $\mathcal{O}$, eq. (1.8), we define the T-odd observables (up-down asymmetries):

$$
\begin{equation*}
A_{T} \equiv \frac{\int d \Omega \operatorname{sgn}(\mathcal{O}) d \Gamma / d \Omega}{\int d \Omega d \Gamma / d \Omega}=\frac{N[\mathcal{O}>0]-N[\mathcal{O}<0]}{N[\mathcal{O}>0]+N[\mathcal{O}<0]} \tag{1.9}
\end{equation*}
$$

where $d \Gamma$ stands for the differential decay width and $d \Omega$ involves the angles of integration. The left hand side of eq. (1.9) shows how the asymmetries are calculated, whereas the right hand side exemplifies how they are measured in experiment: $N[\mathcal{O}>(<) 0]$ is the number of events for which $\mathcal{O}>(<) 0$.

The paper is organized as follows. In section 2 we present the results of our calculations in compact form using the formalism of 14. We propose several T-odd asymmetries in section 3 and point out how the corresponding CP asymmetries can be obtained. In section 4 we perform a numerical analysis of the CP asymmetries proposed and estimate their observability. Finally, we summarize in section 5 .

## 2. Formalism

In order to obtain analytic expressions for the sequential processes (1.1)-(1.7) we shall use the formalism of Kawasaki, Shirafuji and Tsai [14]. According to that formalism the differential decay rates of (1.1)-(1.7), when spin-spin correlations are taken into account, can be written as

$$
\begin{equation*}
d \Gamma=d \Gamma\left(\tilde{b}_{m} \rightarrow t \tilde{\chi}_{j}^{-}\right) \frac{E_{t}}{m_{t} \Gamma_{t}} d \Gamma(t \rightarrow \ldots) \frac{E_{\chi_{j}}}{m_{\chi_{j}} \Gamma_{\chi_{j}}} d \Gamma\left(\tilde{\chi}_{j}^{-} \rightarrow \ldots\right) \tag{2.1}
\end{equation*}
$$

where $d \Gamma(t \rightarrow \ldots)$ and $d \Gamma\left(\tilde{\chi}_{j}^{-} \rightarrow \ldots\right)$ are the differential decay rates of the unpolarized top and unpolarized chargino. The factors $E_{\chi_{j}} /\left(m_{\chi_{j}} \Gamma_{\chi_{j}}\right)$ and $E_{t} /\left(m_{t} \Gamma_{t}\right)$ stem from the narrow width approximation used for $t$ and $\tilde{\chi}_{j}^{-}, \Gamma_{t}$ and $\Gamma_{\chi_{j}}$ are the total widths of $t$ and $\tilde{\chi}_{j}^{-}$, and $\left(E_{t}, m_{t}\right)$ and $\left(E_{\chi_{j}}, m_{\chi_{j}}\right)$ are their energies and masses, respectively. $d \Gamma\left(\tilde{b}_{m} \rightarrow t \tilde{\chi}_{j}^{-}\right)$ is the differential decay rate of the scalar bottom $\tilde{b}_{m}$ into a top quark with the polarization 4 -vector $\xi_{t}^{\alpha}$ and a chargino with the polarization 4 -vector $\xi_{\chi_{j}}^{\alpha}$.

In the scalar bottom rest frame, we have:

$$
\begin{equation*}
d \Gamma\left(\tilde{b}_{m} \rightarrow t \tilde{\chi}_{j}^{-}\right)=\frac{2}{m_{\tilde{b}_{m}}}|A|^{2} d \Phi_{\tilde{b}_{m}} \tag{2.2}
\end{equation*}
$$

where $m_{\tilde{b}_{m}}$ is the mass of the decaying scalar bottom and the phase space element $\Phi_{\tilde{b}}$ is given in eq. (C.1) in appendix G. For the matrix element $A$ we have

$$
\begin{equation*}
A=g \bar{u}\left(p_{t}\right)\left(k_{m j}^{\tilde{b}} P_{L}+l_{m j}^{\tilde{b}} P_{R}\right) v\left(p_{\chi_{j}}\right), \tag{2.3}
\end{equation*}
$$

where $P_{L, R}=\frac{1}{2}\left(1 \mp \gamma_{5}\right), g$ is the $\mathrm{SU}(2)$ gauge coupling constant and the couplings are given in eq. (B.7) in appendix B. For the evaluation of $|A|^{2}$ we use the spin density matrices of $t$ and $\tilde{\chi}_{j}^{-}$:

$$
\begin{equation*}
\rho\left(p_{t}\right)=\Lambda\left(p_{t}\right) \frac{1+\gamma_{5} \not \text { \&t }_{t}}{2}, \quad \rho\left(-p_{\chi_{j}}\right)=-\Lambda\left(-p_{\chi_{j}}\right) \frac{1+\gamma_{5} \not \chi_{\chi_{j}}}{2}, \tag{2.4}
\end{equation*}
$$

with

$$
\begin{equation*}
\Lambda\left(p_{t}\right)=\not p_{t}+m_{t}, \quad \Lambda\left(p_{\chi_{k}}\right)=\not \chi_{\chi_{j}}+m_{\chi_{j}} . \tag{2.5}
\end{equation*}
$$

The matrix element squared is then given by

$$
\begin{align*}
|A|^{2}=\frac{g^{2}}{2}\{ & \left(\left|l_{m j}^{\tilde{b}}\right|^{2}+\left|k_{m j}^{\tilde{b}}\right|^{2}\right)\left[\left(p_{\chi_{j}} p_{t}\right)+m_{\chi_{j}} m_{t}\left(\xi_{\chi_{j}} \xi_{t}\right)\right] \\
& -\left(\left|l_{m j}^{\tilde{b}}\right|^{2}-\left|k_{m j}^{\tilde{b}}\right|^{2}\right)\left[m_{t}\left(p_{\chi_{j}} \xi_{t}\right)+m_{\chi_{j}}\left(\xi_{\chi_{j}} p_{t}\right)\right] \\
& -2 \Re e\left(l_{m j}^{\tilde{b} *} k_{m j}^{\tilde{b}}\right)\left[m_{\chi_{j}} m_{t}-\left(p_{\chi_{j}} \xi_{t}\right)\left(\xi_{\chi_{j}} p_{t}\right)+\left(p_{\chi_{j}} p_{t}\right)\left(\xi_{\chi_{j}} \xi_{t}\right)\right] \\
& \left.+2 \Im m\left(l_{m j}{ }_{m j}^{\tilde{b}}\right) \varepsilon^{\alpha \beta \gamma \delta} p_{\chi_{j} \alpha} \xi_{\chi_{j} \beta} \xi_{t \gamma} p_{t \delta}\right\}, \tag{2.6}
\end{align*}
$$

where we use the convention $\varepsilon^{0123}=1$. The covariant product $\varepsilon^{\alpha \beta \gamma \delta} p_{\chi_{j} \alpha} \xi_{\chi_{j} \beta} \xi_{t \gamma} p_{t \delta}$ in eq. (2.6) contains the triple products which give rise to the CP asymmtries that we study in this paper. Such a term arises due to the interference of the two parts of the amplitude, eq. (2.3), with opposite chiralities proportinal to $k_{m j}^{\tilde{b}}$ and to $l_{m j}^{\tilde{b}}$. The polarization 4vector $\xi_{t}$ is determined through the top quark decays (1.2) and the polarization 4 -vector $\xi_{\chi_{j}}$ is determined through the $\tilde{\chi}_{j}$ decays (1.3)-(1.7). In the following we calculate the polarization 4 -vectors $\xi_{t}$ and $\xi_{\chi_{j}}$ as well as the differential decay rates of $t$ and $\tilde{\chi}_{j}^{-}$for their various decays (1.2) and (1.3)-(1.7). Some of the calculations are quite analogous to those carried out in [13] and in these cases we present the results only.
2.1 Decay rates for $\tilde{\chi}_{j}^{-} \rightarrow \ell_{1}^{-} \overline{\tilde{\nu}} \rightarrow \ell_{1}^{-} \bar{\nu} \tilde{\chi}_{1}^{0}$

The polarization vector of the top quark was obtained in 13] and here we present the results for completeness. The polarization 4 -vector of the top quark determined through the decay $t \rightarrow b W^{+}$, that we shall denote by $\xi_{b}$, equals

$$
\begin{equation*}
\xi_{b}^{\alpha}=\alpha_{b} \frac{m_{t}}{\left(p_{t} p_{b}\right)}\left(p_{b}^{\alpha}-\frac{\left(p_{t} p_{b}\right)}{m_{t}^{2}} p_{t}^{\alpha}\right), \quad \alpha_{b}=\frac{m_{t}^{2}-2 m_{W}^{2}}{m_{t}^{2}+2 m_{W}^{2}} . \tag{2.7}
\end{equation*}
$$

For the polarization vector of the top quark determined in $t \rightarrow b W^{+} \rightarrow b l^{+} \nu$ (and equivalently for $t \rightarrow b W^{+} \rightarrow b c \bar{s}$, where we substitute the the $c$ quark for the lepton), that we denote by $\xi_{l}$, we have

$$
\begin{equation*}
\xi_{l}^{\alpha}=\alpha_{l} \frac{m_{t}}{\left(p_{t} p_{l}\right)}\left(p_{l}^{\alpha}-\frac{\left(p_{t} p_{l}\right)}{m_{t}^{2}} p_{t}^{\alpha}\right), \quad \alpha_{l}=-1 \tag{2.8}
\end{equation*}
$$

The polarization vector of $\tilde{\chi}_{j}^{-}$is determined solely through the decay $\tilde{\chi}_{j}^{-} \rightarrow \ell_{1}^{-} \overline{\tilde{\nu}}$, as the subsequent decay of $\tilde{\nu}$, being a scalar particle, does not contribute. We obtain:

$$
\begin{equation*}
\xi_{\chi_{j}}^{\alpha}=\alpha_{\tilde{\nu}} \frac{m_{\chi_{j}}}{\left(p_{\chi_{j}} p_{\ell_{1}}\right)}\left(p_{\ell_{1}}^{\alpha}-\frac{\left(p_{\chi_{j}} p_{\ell_{1}}\right)}{m_{\chi_{j}}^{2}} p_{\chi_{j}}^{\alpha}\right), \quad \alpha_{\tilde{\nu}}=\frac{\left|l_{j}^{\tilde{\nu}}\right|^{2}-\left|k_{j}^{\tilde{\nu}}\right|^{2}}{\left|l_{j}^{\tilde{j}}\right|^{2}+\left|k_{j}^{\tilde{j}}\right|^{2}} . \tag{2.9}
\end{equation*}
$$

Further, according to eq. (2.1), we need the differential decay rates of $t$ and $\tilde{\chi}_{j}^{-}$. The distribution of the leptons in the sequential decay (1.3), in the narrow width approximation for $\tilde{\nu}$, is given by

$$
\begin{equation*}
d \Gamma_{\chi_{j}}^{\mathrm{I}}\left(\tilde{\chi}_{j}^{-} \rightarrow \ell_{1}^{-} \bar{\nu} \tilde{\chi}_{1}^{0}\right)=d \Gamma\left(\tilde{\chi}_{j}^{-} \rightarrow \tilde{\ell}_{1}^{-} \overline{\tilde{\nu}}\right) B R\left(\overline{\tilde{\nu}} \rightarrow \bar{\nu} \tilde{\chi}_{1}^{0}\right), \tag{2.10}
\end{equation*}
$$

where $B R\left(\overline{\tilde{\nu}} \rightarrow \bar{\nu} \tilde{\chi}_{1}^{0}\right)$ is the branching ratio of the decay $\overline{\tilde{\nu}} \rightarrow \bar{\nu} \tilde{\chi}_{1}^{0}$ and

$$
\begin{equation*}
d \Gamma\left(\tilde{\chi}_{j}^{-} \rightarrow \ell_{1}^{-} \overline{\tilde{\nu}}\right)=\frac{g^{2}\left(\left|k_{j}^{\tilde{\nu}}\right|^{2}+\left|l_{j}^{\tilde{\nu}}\right|^{2}\right)\left(p_{\chi_{j}} p_{\ell_{1}}\right)}{2 E_{\chi_{j}}} d \Phi_{\chi_{j}}^{1}, \tag{2.11}
\end{equation*}
$$

where the couplings are given in eq. (B.8) in appendix B and the phase space element $d \Phi_{\chi_{j}}^{1}$ is given in eq. (C.5) in appendix $\mathbb{G}$. The differential decay rates of the top quark are (see for instance [13]):

$$
\begin{gather*}
d \Gamma\left(t \rightarrow b W^{+}\right)=\frac{g^{2}\left(m_{t}^{2}-m_{W}^{2}\right)\left(2 m_{W}^{2}+m_{t}^{2}\right)}{8 E_{t} m_{W}^{2}} d \Phi_{t}^{b},  \tag{2.12}\\
d \Gamma\left(t \rightarrow b l^{+} \nu\right)=\frac{g^{4} \pi\left(p_{t} p_{l}\right)\left(m_{t}^{2}-2\left(p_{t} p_{l}\right)\right)}{2 E_{t} m_{W} \Gamma_{W}} d \Phi_{t}^{l} \tag{2.13}
\end{gather*}
$$

with $d \Phi_{t}^{b, l}$ given in eqs. (C.2) and (C.3) in appendix G.
The angular distributions of the decay products of $t$ and $\tilde{\chi}_{j}^{-}$decay mode (1.3) are obtained by inserting the differential decay rate of the scalar bottom, eq. (2.2), the differential decay rates of the top quark, eqs. (2.12) and (2.13), and the differential decay rate of the chargino, eq. (2.10), into eq. (2.1), where we use the appropriate polarization vectors as given in eqs. (2.7)-(2.9). The differential decay rates of $\tilde{b}_{m}$ then read

$$
\begin{align*}
d \Gamma_{f}^{\mathrm{I}}= & N_{f} \frac{g^{6} B R\left(\tilde{\nu} \rightarrow \nu \tilde{\chi}_{1}^{0}\right)\left(p_{\chi_{j}} p_{\ell_{1}}\right)\left(\left|\nu_{j}^{\tilde{\nu}}\right|^{2}+\left|k_{j}^{\tilde{\nu}}\right|^{2}\right)}{8 m_{\tilde{b}} m_{t} \Gamma_{t} m_{\chi_{j}} \Gamma_{\chi_{j}}} \\
& \times\left\{\left(\left|l_{m j}^{\tilde{b}}\right|^{2}+\left|k_{m j}^{\tilde{b}}\right|^{2}\right)\left(p_{\chi_{j}} p_{t}\right)-2 \Re e\left(l_{m j}^{\tilde{b}} k_{m j}^{\tilde{b}}\right) m_{\chi_{j}} m_{t}+\cdots\right. \\
& \left.+2 \Im m\left(l_{m j}^{\tilde{b} *} k_{m j}^{\tilde{b}}\right) \alpha_{f} \alpha_{\tilde{\nu}} \frac{m_{t}}{\left(p_{t} p_{f}\right)} \frac{m_{\chi_{j}}}{\left(p_{\chi_{j}} p_{\ell_{1}}\right)} m_{\tilde{b}}\left(\mathbf{p}_{\ell_{1}} \mathbf{p}_{f} \mathbf{p}_{t}\right)\right\} d \Phi_{f}^{\mathrm{I}}, \tag{2.14}
\end{align*}
$$

where the subindex $f=b, l$ is to distinguish the two $t$ quark decays in (1.2). The prefactors in eq. (2.14) are

$$
\begin{align*}
& N_{b}=\frac{\left(m_{t}^{2}-m_{W}^{2}\right)\left(2 m_{W}^{2}+m_{t}^{2}\right)}{2 m_{W}^{2}}, \\
& N_{l}=\frac{g^{2} 2 \pi\left(p_{t} p_{l}\right)\left(m_{t}^{2}-2\left(p_{t} p_{l}\right)\right)}{m_{W} \Gamma_{W}}, \tag{2.15}
\end{align*}
$$

and the phase space elements equal

$$
\begin{equation*}
d \Phi_{f}^{\mathrm{I}}=d \Phi_{\tilde{b}_{m}} d \Phi_{t}^{f} d \Phi_{\chi_{j}}^{1} . \tag{2.16}
\end{equation*}
$$

In eq. (2.14) we have only included those terms which are needed for the calculation of the up-down asymmetries in eq. (1.9). The omitted terms, represented by dots, are T-even and thus, cannot contribute to the numerator of eq. (1.9). Further, as they depend on the polarizations of either the top quark or the chargino, they cannot contribute to the denominator of eq. (1.9).

### 2.2 Decay rates for $\tilde{\chi}_{j}^{-} \rightarrow \tilde{\ell}_{n}^{-} \bar{\nu} \rightarrow \ell_{2}^{-} \bar{\nu} \tilde{\chi}_{1}^{0}$

In order to obtain the angular correlations among the $t$ decay products and the lepton $\ell_{2}$ stemming from the $\tilde{\chi}_{j}^{-}$decay (1.4), we need the polarization 4 -vector of $\tilde{\chi}_{j}^{-}$determined in the decay (1.4). As $\tilde{\ell}_{n}$ is a scalar particle, $\xi_{\chi_{j}}$ is determined solely in the decay $\tilde{\chi}_{j}^{-} \rightarrow \tilde{\ell}_{n}^{-} \bar{\nu}$. We obtain:

$$
\begin{equation*}
\xi_{\chi_{j}}^{\alpha}=\alpha_{\tilde{\ell}} \frac{m_{\chi_{j}}}{\left(p_{\chi_{j}} p_{\nu}\right)}\left(p_{\nu}^{\alpha}-\frac{\left(p_{\chi_{j}} p_{\nu}\right)}{m_{\chi_{j}}^{2}} p_{\chi_{j}}^{\alpha}\right), \quad \alpha_{\tilde{\ell}}=-1 . \tag{2.17}
\end{equation*}
$$

The differential decay rate of the decay chain (1.4), in the narrow width approximation for $\tilde{\ell}_{n}^{-}$, reads

$$
\begin{equation*}
d \Gamma_{\chi_{j}}^{\mathrm{II}}\left(\tilde{\chi}_{j}^{-} \rightarrow \ell_{2}^{-} \bar{\nu} \tilde{\chi}_{1}^{0}\right)=d \Gamma\left(\tilde{\chi}_{j}^{-} \rightarrow \tilde{\ell}_{n}^{-} \bar{\nu}\right) \frac{E_{\tilde{\ell}}}{m_{\tilde{\ell}} \Gamma_{\tilde{\ell}}} d \Gamma\left(\tilde{\ell}_{n}^{-} \rightarrow \tilde{\chi}_{1}^{0} \ell_{2}^{-}\right), \tag{2.18}
\end{equation*}
$$

with the differential decay rates for $\tilde{\chi}_{j}^{-} \rightarrow \tilde{\ell}_{n}^{-} \bar{\nu}$ and $\tilde{\ell}_{n} \rightarrow \tilde{\chi}_{1}^{0} \ell_{2}^{-}$given by

$$
\begin{equation*}
d \Gamma\left(\tilde{\chi}_{j}^{-} \rightarrow \tilde{\ell}_{n}^{-} \bar{\nu}\right)=\frac{g^{2}}{2 E_{\chi_{j}}}\left|l_{n j}^{\tilde{\ell}}\right|^{2}\left(p_{\chi_{j}} p_{\nu}\right) d \Phi_{\chi_{j}}^{2} \tag{2.19}
\end{equation*}
$$

and

$$
\begin{equation*}
d \Gamma\left(\tilde{\ell}_{n}^{-} \rightarrow \tilde{\chi}_{1}^{0} \ell_{2}^{-}\right)=\frac{g^{2}}{E_{\tilde{\ell}}}\left(\left|a_{n k}\right|^{2}+\mid b_{n k} \tilde{\ell}^{2}\right)\left(p_{\tilde{\ell}} p_{\ell_{2}}\right) d \Phi_{\tilde{\ell}}, \tag{2.20}
\end{equation*}
$$

where the couplings are given in appendix $G$ in eqs. (B.9) and (B.11). The phase space elements $d \Phi_{\chi_{j}}^{2}$ and $d \Phi_{\tilde{\ell}}$ are given in appendix B in eqs. (C.6) and (C.7), respectively.

The angular distributions of the decay products of $t$ are the same as in the previous case. On the other hand, the angular distribution of the decay products of the $\tilde{\chi}_{j}^{-}$decay mode (1.4) is given by eq. (2.18) which we insert into eq. (2.1) in order to obtain the differential decay rates of the combined process (1.1), (1.2) and (1.4). The polarization vector of the chargino is determined through the decay (1.4) and is given in eq. (2.17). Then the differential decay rates of $\tilde{b}_{m}$ read

$$
\begin{align*}
d \Gamma_{f}^{\mathrm{II}}= & N_{f} \frac{\left.g^{8}\left(p_{\chi_{j}} p_{\nu}\right)\left(p_{\chi_{1}^{0}} p_{\ell_{2}}\right)| |_{n j}^{\tilde{\ell}}\right|^{2}\left(\left|a_{n k}^{\tilde{\ell}}\right|^{2}+\left|b_{n k}^{\tilde{\ell}}\right|^{2}\right)}{8 m_{\tilde{b}} m_{t} \Gamma_{t} m_{\chi_{j}} \Gamma_{\chi_{j}} m_{\tilde{\ell}} \Gamma_{\tilde{\ell}}} \\
& \times\left\{\left(\left|l_{m j}^{\tilde{b}}\right|^{2}+\left|k_{m j}^{\tilde{b}}\right|^{2}\right)\left(p_{\chi_{j}} p_{t}\right)-2 \Re e\left(l_{m j}^{\tilde{b} *} k_{m j}^{\tilde{b}}\right) m_{\chi_{j}} m_{t}+\cdots\right. \\
& \left.+2 \Im m\left(l_{m j}^{\tilde{b} *} k_{m j}^{\tilde{b}}\right) \alpha_{f} \alpha_{\tilde{\ell}} \frac{m_{t}}{\left(p_{t} p_{f}\right)} \frac{m_{\chi_{j}}}{\left(p_{\chi_{j}} p_{\nu}\right)} m_{\tilde{b}}\left(\mathbf{p}_{\ell_{2}} \mathbf{p}_{f} \mathbf{p}_{t}\right)\right\} d \Phi_{f}^{\mathrm{II}}, \tag{2.21}
\end{align*}
$$

where the phase space elements equal

$$
\begin{equation*}
d \Phi_{f}^{\mathrm{II}}=d \Phi_{\tilde{b}_{m}} d \Phi_{t}^{f} d \Phi_{\chi_{j}}^{2} d \Phi_{\tilde{\ell}} \tag{2.22}
\end{equation*}
$$

As in the previous case, we have omitted those terms in eq. (2.21) (denoted by dots) which are unessential for the calculation of the up-down asymmetries, eq. (1.9).

### 2.3 Decay rates for $\tilde{\chi}_{j}^{-} \rightarrow W^{-} \tilde{\chi}_{1}^{0} \rightarrow \ell_{3}^{-} \bar{\nu} \tilde{\chi}_{1}^{0}$

When the decay of $\tilde{\chi}_{j}^{-}$proceeds via the $W^{-}$boson exchange, (1.5), the polarization 4 -vector $\xi_{\chi_{j}}$ is parameterized by two components that are in the $\tilde{\chi}_{j}^{-}$decay plane and a component normal to it. It can be written completely general as

$$
\begin{equation*}
\xi_{\chi_{j}}^{\alpha}=P_{\ell} Q_{\ell}^{\alpha}+P_{\nu} Q_{\nu}^{\alpha}+D^{C P} \varepsilon^{\alpha \beta \gamma \delta} p_{\ell_{3} \beta} p_{\nu \gamma} p_{\chi_{j} \delta} \tag{2.23}
\end{equation*}
$$

where the 4 -vectors $Q_{\ell}^{\alpha}$ and $Q_{\nu}^{\alpha}$ are in the decay plane of $\tilde{\chi}_{j}^{-}$:

$$
\begin{equation*}
Q_{\ell}^{\alpha}=p_{\ell_{3}}^{\alpha}-\frac{\left(p_{\ell_{3}} p_{\chi_{j}}\right)}{m_{\chi_{j}}^{2}} p_{\chi_{j}}^{\alpha}, \quad Q_{\nu}^{\alpha}=p_{\nu}^{\alpha}-\frac{\left(p_{\nu} p_{\chi_{j}}\right)}{m_{\chi_{j}}^{2}} p_{\chi_{j}}^{\alpha} \tag{2.24}
\end{equation*}
$$

and $\varepsilon^{\alpha \beta \gamma \delta} p_{\ell_{3} \beta} p_{\nu \gamma} p_{\chi_{j} \delta}$ is orthogonal to it. For the components in the decay plane we obtain

$$
\begin{align*}
& P_{\ell}=\frac{m_{\chi_{j}}\left|O_{1 j}^{L}\right|^{2}\left(2\left(p_{\nu} p_{\chi_{j}}\right)-m_{W}^{2}\right)-2 m_{\chi_{1}^{0}}\left(p_{\nu} p_{\chi_{j}}\right) \Re e\left(O_{1 j}^{L *} O_{1 j}^{R}\right)}{|C|^{2}}, \\
& P_{\nu}=\frac{-m_{\chi_{j}}\left|O_{1 j}^{R}\right|^{2}\left(2\left(p_{\ell_{3}} p_{\chi_{j}}\right)-m_{W}^{2}\right)+2 m_{\chi_{1}^{0}}\left(p_{\ell_{3}} p_{\chi_{j}}\right) \Re e\left(O_{1 j}^{L *} O_{1 j}^{R}\right)}{|C|^{2}}, \tag{2.25}
\end{align*}
$$

with

$$
\begin{align*}
|C|^{2}= & -m_{W}^{2}\left[\left|O_{1 j}^{L}\right|^{2}\left(p_{\ell_{3}} p_{\chi_{j}}\right)+\left|O_{1 j}^{R}\right|^{2}\left(p_{\nu} p_{\chi_{j}}\right)+m_{\chi_{1}^{0}} m_{\chi_{j}} \Re e\left(O_{1 j}^{L *} O_{1 j}^{R}\right)\right] \\
& +2\left(p_{\ell_{3}} p_{\chi_{j}}\right)\left(p_{\nu} p_{\chi_{j}}\right)\left(\left|O_{1 j}^{L}\right|^{2}+\left|O_{1 j}^{R}\right|^{2}\right) \tag{2.26}
\end{align*}
$$

where the couplings are given in appendix B in eq. (B.12). The component normal to the decay plane reads

$$
\begin{equation*}
D^{C P}=\frac{2 m_{\chi_{1}^{0}} \Im m\left(O_{1 j}^{L *} O_{1 j}^{R}\right)}{|C|^{2}} \tag{2.27}
\end{equation*}
$$

The component $D^{C P}$ is sensitive to CP violation in the $\tilde{\chi}_{j}^{-} \tilde{\chi}_{1}^{0} W^{+}$couplings, i.e. to the phases $\phi_{\mu}$ and $\phi_{M_{1}}$. The decay rate distribution of $\tilde{\chi}_{j}^{-} \rightarrow W^{-} \tilde{\chi}_{1}^{0} \rightarrow \ell_{3}^{-} \bar{\nu} \tilde{\chi}_{k}^{0}$ is given by

$$
\begin{equation*}
d \Gamma_{\chi_{j}}^{\mathrm{III}}\left(\tilde{\chi}_{j}^{-} \rightarrow \ell_{3}^{-} \bar{\nu} \tilde{\chi}_{1}^{0}\right)=\sum_{ \pm} \frac{g^{4} \pi}{m_{W} \Gamma_{W} E_{\chi_{j}}}|C|^{2} d \Phi_{\chi_{j}}^{\mathrm{III}} \tag{2.28}
\end{equation*}
$$

where $d \Phi_{\chi_{j}}^{\mathrm{III}}=\frac{1}{2 \pi}\left(d \Phi_{\chi_{j}}^{3}\right)^{ \pm} d \Phi_{W}^{3}$ with $\left(d \Phi_{\chi_{j}}^{3}\right)^{ \pm}$being the phase space element for the decay $\tilde{\chi}_{j}^{-} \rightarrow W^{-} \tilde{\chi}_{1}^{0}$, eq. (C.8) in appendix $\mathbb{Q}$, and $d \Phi_{W}^{3}$ is the phase space element for the decay $W^{-} \rightarrow \ell_{3}^{-} \bar{\nu}$, eq. (С.11).

The angular distributions of the decay products of $\tilde{b}_{m}$, where the chargino decays according to (1.5), can now be obtained in the same manner as in the previous two cases. Again we only quote the terms that are essential for the calculation of the up-down asymmetries in eq. (1.9):

$$
\begin{align*}
d \Gamma_{f}^{\mathrm{III}}= & \sum_{ \pm} N_{f} \frac{g^{8} \pi|C|^{2}}{4 m_{\tilde{b}} m_{t} \Gamma_{t} m_{\chi_{j}} \Gamma_{\chi_{j}} m_{W} \Gamma_{W}} \\
& \times\left\{\left(\left|l_{m j}^{\tilde{b}}\right|^{2}+\left|k_{m j}^{\tilde{b}}\right|^{2}\right)\left(p_{\chi_{j}} p_{t}\right)-2 \Re e\left(l_{m j}^{\tilde{b} *} k_{m j}^{\tilde{b}}\right) m_{\chi_{j}} m_{t}+\cdots\right. \\
& \left.\left.+2 \alpha_{f} \Im m\left(l_{m j}^{\tilde{b} *} k_{m j}^{\tilde{b}}\right) \frac{m_{t}}{\left(p_{t} p_{f}\right)}\left(P_{\ell}-P_{\nu}\right) m_{\tilde{b}}\left(\mathbf{p}_{\ell_{3}} \mathbf{p}_{f} \mathbf{p}_{t}\right)\right]\right\} d \Phi_{f}^{\mathrm{III}}, \tag{2.29}
\end{align*}
$$

with

$$
\begin{equation*}
d \Phi_{f}^{\mathrm{III}}=d \Phi_{\tilde{b}} d \Phi_{t}^{f} d \Phi_{\chi_{j}}^{\mathrm{III}} \tag{2.30}
\end{equation*}
$$

where the sum in eqs. $(\overline{2.28})$ and $(\sqrt[2.29]{ })$ corresponds to the two kinematical solutions for $E_{\ell_{3}}$ (for details see appendix ${ }^{[ }$).

In principle, the normal component of the chargino polarization vector in eq. (2.23) will also give rise to triple products proportional to $\Im m\left(O_{k j}^{L}{ }^{*} O_{k j}^{R}\right)$. However, in order to be sensitive to these triple products, the reconstruction of the decay plane of the chargino would be necessary. In practice, this cannot be accomplished, because the neutrino as well as the neutralino escape detection in experiment.

### 2.4 Decay rates for $\tilde{\chi}_{j}^{-} \rightarrow W^{-} \tilde{\chi}_{1}^{0}$

Finally we consider the two-body decay mode of $\tilde{\chi}_{j}^{-}$(1.7). The polarization 4 -vector of $\tilde{\chi}_{j}^{-}$ in this case is given as

$$
\begin{equation*}
\xi_{\chi_{j}}^{\alpha}=\alpha_{W} \frac{m_{\chi_{j}}}{\left(p_{\chi_{j}} p_{W}\right)}\left(p_{W}^{\alpha}-\frac{\left(p_{W} p_{\chi_{j}}\right)}{m_{\chi_{j}}^{2}} p_{\chi_{j}}^{\alpha}\right), \tag{2.31}
\end{equation*}
$$

with

$$
\begin{equation*}
\alpha_{W}=2\left(\frac{\left|O_{1 j}^{L}\right|^{2}-\left|O_{1 j}^{R}\right|^{2}}{\left|C_{W}\right|^{2}}\right)\left(\frac{m_{\chi_{j}}^{2}-2 m_{W}^{2}-m_{\chi_{1}^{0}}^{2}}{m_{W}^{2}}\right)\left(p_{\chi_{j}} p_{W}\right), \tag{2.32}
\end{equation*}
$$

where

$$
\begin{align*}
\left|C_{W}\right|^{2}= & \left(\left|O_{1 j}^{L}\right|^{2}+\left|O_{1 j}^{R}\right|^{2}\right)\left[\frac{\left(m_{\chi_{1}^{0}}^{2}+m_{\chi_{j}}^{2}\right) m_{W}^{2}+\left(m_{\chi_{1}^{0}}^{2}-m_{\chi_{j}}^{2}\right)^{2}-2 m_{W}^{4}}{m_{W}^{2}}\right] \\
& -12 \Re e\left(O_{1 j}^{L *} O_{1 j}^{R}\right) m_{\chi_{1}^{0}} m_{\chi_{j}} . \tag{2.33}
\end{align*}
$$

The decay rate distribution of $\tilde{\chi}_{j}^{-} \rightarrow W^{-} \tilde{\chi}_{1}^{0}$ is given by

$$
\begin{equation*}
d \Gamma_{\chi_{j}}^{W}\left(\tilde{\chi}_{j}^{-} \rightarrow W^{-} \tilde{\chi}_{1}^{0}\right)=\sum_{ \pm} \frac{g^{2}}{4 E_{\chi_{j}}}\left|C_{W}\right|^{2}\left(d \Phi_{\chi_{j}}^{3}\right)^{ \pm} . \tag{2.34}
\end{equation*}
$$

The angular distribution of the decay products of $\tilde{b}_{m}$, with the chargino two-body decay (1.7), is given by

$$
\begin{align*}
d \Gamma_{f}^{W}= & \sum_{ \pm} N_{f} \frac{g^{6}\left|C_{W}\right|^{2}}{16 m_{\tilde{b}} m_{t} \Gamma_{t} m_{\chi_{j}} \Gamma_{\chi_{j}}} \\
& \times\left\{\left(\left|l_{m j}^{\tilde{b}}\right|^{2}+\left|k_{m j}^{\tilde{b}}\right|^{2}\right)\left(p_{\chi_{j}} p_{t}\right)-2 \Re e\left(l_{m j}^{\tilde{b} *} k_{m j}^{\tilde{b}}\right) m_{\chi_{j}} m_{t}+\cdots\right. \\
& \left.+2 \Im m\left(l_{m j}^{\tilde{b} *} k_{m j}^{\tilde{b}}\right) \alpha_{f} \alpha_{W} \frac{m_{t}}{\left(p_{t} p_{f}\right)} \frac{m_{\chi_{j}}}{\left(p_{\chi_{j}} p_{\ell_{1}}\right)} m_{\tilde{b}}\left(\mathbf{p}_{W} \mathbf{p}_{f} \mathbf{p}_{t}\right)\right\} d \Phi_{f}^{W}, \tag{2.35}
\end{align*}
$$

with

$$
\begin{equation*}
d \Phi_{f}^{W}=d \Phi_{\tilde{b}} d \Phi_{t}^{f}\left(d \Phi_{\chi_{j}}^{3}\right)^{ \pm} \tag{2.36}
\end{equation*}
$$

where again we have quoted only the terms that contribute to the up-down asymmetries in eq. (1.9). The sum in eq. (2.35) is due to the two kinematical solutions for $\left|\mathbf{p}_{W}\right|$ (for details see appendix (G).

## 3. T-odd asymmetries

A general definition of the T-odd observables which we study in this paper has been given in eq. (1.9). For the following it is convenient to introduce a shorthand notation for the various T-odd asymmetries to be studied below:

$$
\begin{equation*}
A_{i j k}=\frac{N\left[\left(\mathbf{p}_{i} \mathbf{p}_{j} \mathbf{p}_{k}\right)>0\right]-N\left[\left(\mathbf{p}_{i} \mathbf{p}_{j} \mathbf{p}_{k}\right)<0\right]}{N\left[\left(\mathbf{p}_{i} \mathbf{p}_{k} \mathbf{p}_{k}\right)>0\right]+N\left[\left(\mathbf{p}_{i} \mathbf{p}_{j} \mathbf{p}_{k}\right)<0\right]}, \tag{3.1}
\end{equation*}
$$

where $N\left[\left(\mathbf{p}_{i} \mathbf{p}_{j} \mathbf{p}_{k}\right)>0\right]\left(N\left[\left(\mathbf{p}_{i} \mathbf{p}_{j} \mathbf{p}_{k}\right)<0\right]\right)$ are the number of events with $\left(\mathbf{p}_{i} \mathbf{p}_{j} \mathbf{p}_{k}\right)>0$ $\left(\left(\mathbf{p}_{i} \mathbf{p}_{j} \mathbf{p}_{k}\right)<0\right)$. The indices $i, j, k$ specify the observed particles appearing in the considered decay mode of $\tilde{b}_{m}$. We choose the convention that $\mathbf{p}_{i}$ denotes the momentum of a particle originating from the $\tilde{\chi}_{j}^{-}$decay, $\mathbf{p}_{j}$ denotes the momentum of a fermion from the top quark decay and $\mathbf{p}_{k}$ either denotes the momentum of the top quark itself or of another particle stemming from the decay of the top quark. According to the different decay channels we group the considered triple products as follows:

1. If the 3 -body decay $\tilde{\chi}_{j}^{-} \rightarrow \ell_{i}^{-} \bar{\nu} \tilde{\chi}_{1}^{0}$ is considered, the only detectable particles are the final charged leptons $\ell_{1}^{-}, \ell_{1}^{-}, \ell_{3}^{-}$. We shall distinguish two classes of asymmetries depending on whether the leptons $\ell_{1}^{-}, \ell_{2}^{-}, \ell_{3}^{-}$, originating from the different decay chains (1.3) $-(1.5)$, are distinguishable or not.
(a) First we define the T-odd asymmetries where the leptons from the $\tilde{\chi}_{j}^{-}$decays (1.3)-(1.5) can be distinguished. ${ }^{1}$ The triple products on which the T-odd

[^0]asymmetries are based in this case, are given by
\[

$$
\begin{align*}
& \left(\mathbf{p}_{\ell_{i}}-\mathbf{p}_{b} \mathbf{p}_{t}\right) \text { and } \quad\left(\mathbf{p}_{\ell_{i}} \mathbf{p}_{b} \mathbf{p}_{W^{+}}\right), \quad \text { when } \quad t \rightarrow b W^{+} \rightarrow b q \bar{q}^{\prime},  \tag{3.2}\\
& \left(\mathbf{p}_{\ell_{i}} \mathbf{p}_{l+} \mathbf{p}_{b}\right), \text { when } t \rightarrow b W^{+} \rightarrow b l^{+} \nu,  \tag{3.3}\\
& \left(\mathbf{p}_{\ell_{i}}-\mathbf{p}_{c} \mathbf{p}_{t}\right),\left(\mathbf{p}_{\ell_{i}^{-}} \mathbf{p}_{c} \mathbf{p}_{b}\right) \text { and }\left(\mathbf{p}_{\ell_{i}^{-}} \mathbf{p}_{c} \mathbf{p}_{s}\right), \text { when } t \rightarrow b W^{+} \rightarrow b c \bar{s}, \tag{3.4}
\end{align*}
$$
\]

where for the triple products in (3.4) it is necessary to identify the $c$ quark which is expected to be possible with reasonable efficiency and purity [15-17. With the triple products in (3.2)-(3.4) the associated T-odd asymmetries can be defined according to eq. (3.1), where in the following we use the notation $A_{\ell_{\cdot}^{-} b t}$ and $A_{\ell_{i}^{-} b W^{+}}$for the T-odd asymmetries based on the triple products in (3.2) etc. Note that $A_{\ell_{i}^{-} b t}$ and $A_{\ell_{i}^{-} b W^{+}}$have the same value due to momentum conservation.
(b) We define a second class of T-odd asymmetries where it is not necessary to distinguish the different leptonic $\tilde{\chi}_{j}^{-}$decay chains, eqs. (1.3)-(1.5). This class of T-odd asymmetries is based on the triple products as given in (3.2)-(3.4) where $\ell_{i}^{-}$is replaced by $\ell^{-}$. Then $N\left[\left(\mathbf{p}_{\ell^{-}} \mathbf{p}_{b} \mathbf{p}_{t}\right)>0\right]$ in eq. (3.1) means
$N\left[\left(\mathbf{p}_{\ell^{-}} \mathbf{p}_{b} \mathbf{p}_{t}\right)>0\right]=N\left[\left(\mathbf{p}_{\ell_{1}} \mathbf{p}_{b} \mathbf{p}_{t}\right)>0\right]+N\left[\left(\mathbf{p}_{\ell_{2}} \mathbf{p}_{b} \mathbf{p}_{t}\right)>0\right]+N\left[\left(\mathbf{p}_{\ell_{3}} \mathbf{p}_{b} \mathbf{p}_{t}\right)>0\right]$.
For this class of T-odd asymmetries we will use the notation $A_{\ell^{-} \text {bt }}$ etc. The following formula relates $A_{\ell^{-} j k}$ to the asymmetries $A_{\ell_{i}^{-} j k}$ and the branching ratios $B R_{\ell_{i}} \equiv B R\left(\tilde{\chi}_{j}^{-} \rightarrow \ell_{i}^{-} \bar{\nu} \tilde{\chi}_{1}^{0}\right)$ of the decay chains (1.3)-(1.5):

$$
\begin{equation*}
A_{\ell^{-} j k}=\frac{B R_{\ell_{1}^{-}}}{B R_{\ell^{-}}} A_{\ell_{1}^{-} j k}+\frac{B R_{\ell_{2}^{-}}}{B R_{\ell^{-}}} A_{\ell_{2}^{-} j k}+\frac{B R_{\ell_{3}^{-}}}{B R_{\ell^{-}}} A_{\ell_{3}^{-} j k}, \tag{3.5}
\end{equation*}
$$

where we have introduced the shorthand notation $B R_{\ell^{-}}$:

$$
\begin{equation*}
B R_{\ell^{-}}=B R_{\ell_{1}^{-}}+B R_{\ell_{2}^{-}}+B R_{\ell_{3}^{-}} \tag{3.6}
\end{equation*}
$$

This formula allows us to calculate the contribution of $A_{\ell_{i}^{-} j k}$ to the asymmetry $A_{\ell^{-} j k}$, depending on the branching ratios of the different decay modes of $\tilde{\chi}_{j}^{-}$.
2. If we consider the 2-body decay mode $\tilde{\chi}_{j}^{-} \rightarrow W^{-} \tilde{\chi}_{1}^{0}$, where the $W$ boson decays hadronically so that its momentum vector can be reconstructed, we can define analogous triple products as in (3.2)-(3.4) with $\ell_{i}^{-}$replaced by $W^{-}$. For the corresponding T-odd asymmetries again we use the notation $A_{W^{-b t}}$ etc.
At the end of this section, we discuss how CP-odd asymmetries can be obtained from the T-odd asymmetries defined above. It is well known that a non-zero value of the considered T-odd asymmetries does not necessarily imply that the CP symmetry is violated since final state interactions give rise (although only at loop level) to the same asymmetries. In order to identify a genuine signal of CP violation one needs to consider also the C-conjugate decay. T-odd asymmetries that are based on triple
products analogous to the one given in (3.2)-(3.4) can be defined for the charge conjugate decay $\overline{\tilde{b}}_{m} \rightarrow \tilde{\chi}_{j}^{+} \bar{t}$ as well, and we denote them by $\bar{A}_{i j k}$. One finds that the term of the matrix element squared for the C-conjugate decay $\overline{\tilde{b}}_{m} \rightarrow \tilde{\chi}_{j}^{+} \bar{t}$ that comprises the triple product has the same sign as the corresponding term for the decay $\tilde{b}_{m} \rightarrow \tilde{\chi}_{j}^{-} t$. Thus, true CP violating asymmetries are obtained when summing the T-odd asymmetries that arise in the decays $\tilde{b}_{m} \rightarrow \tilde{\chi}_{j}^{-} t$ and $\tilde{\tilde{b}}_{m} \rightarrow \tilde{\chi}_{j}^{+} \bar{t}$ :

$$
\begin{equation*}
A_{i j k}^{\mathrm{CP}}=\frac{A_{i j k}+\bar{A}_{i j k}}{2} \tag{3.7}
\end{equation*}
$$

## 4. Numerical results

Now we study numerically the CP asymmetries defined in the previous section, where we focus on their dependence on the CP phases, in particular on $\phi_{A_{b}}$. All CP asymmetries defined in the previous section are proportional to $\Im m\left(l_{m j}^{\tilde{b} *} k_{m j}^{\tilde{b}}\right)$, see eq. (2.6). Hence they measure combinations of CP phases in the MSSM. In order to see more easily the dependence of the CP asymmetries on the parameters, it is useful to expand:

$$
\begin{equation*}
\Im m\left(l_{m j}^{\tilde{b} *} k_{m j}^{\tilde{b}}\right)=-Y_{t}\left[c_{m} \Im m\left(V_{j 2}^{*} U_{j 1}^{*}\right)-\frac{1}{2} Y_{b} \sin 2 \theta_{\tilde{b}} d_{m} \Im m\left(V_{j 2}^{*} U_{j 2}^{*} e^{-i \phi_{\tilde{b}}}\right)\right], \tag{4.1}
\end{equation*}
$$

with $Y_{t}$ and $Y_{b}$ the top quark and bottom quark Yukawa couplings, $c_{1}=\cos ^{2} \theta_{\tilde{b}}, c_{2}=\sin ^{2} \theta_{\tilde{b}}$, $d_{1}=1, d_{2}=-1$, and $\theta_{\tilde{b}}$ and $\phi_{\tilde{b}}$ the mixing angle and the CP phase of the scalar bottom system given in appendix A. In general the quantity in eq. (4.1) can be large due to the large $t$ - and $b$-quark Yukawa couplings. The relevant phases are $\phi_{\mu}$ and $\phi_{A_{b}}$. For $\phi_{\mu}=0$, we have $\Im m\left(l_{m j}^{\tilde{b} *} k_{m j}^{\tilde{b}}\right) \propto \sin 2 \theta_{\tilde{b}} \sin \phi_{\tilde{b}}$ and from the explicit expressions given in appendix $\mathbb{A}$, we obtain $\sin 2 \theta_{\tilde{b}} \sin \phi_{\tilde{b}} \propto \sin \phi_{A_{b}}$. As we will see below also the asymmetries show such a $\sin \phi_{A_{b}}$ behavior and thus, their largest values are attained at $\phi_{A_{b}}=\pi / 2,3 \pi / 2$. As $\phi_{\tilde{b}}$ is sensitive to $\phi_{A_{b}}$ if $\left|A_{b}\right| \gtrsim|\mu| \tan \beta$, we need a small value for $\tan \beta$ and a large value for $\left|A_{b}\right|$ compatible with the constraint due to the tree-level vacuum stability condition 18]. Note that in the case where $|\mu| \tan \beta \gg\left|A_{b}\right|$ we have $\sin \phi_{\tilde{b}} \approx 0$ if $\phi_{\mu}=0, \pi$.

For our numerical studies we adopt the two scenarios given in table 1. In the two scenarios we have assumed the gaugino mass relation $\left|M_{1}\right|=5 / 3 \tan ^{2} \Theta_{W} M_{2}$, with $\phi_{M_{1}}=$ 0 , and we have fixed the scalar bottom masses assuming $M_{\tilde{Q}}>M_{\tilde{D}}$. In scenario A the scalar bottom masses are heavy enough to allow for all considered decays of $\tilde{\chi}_{j}^{-}$, eq. (1.3)(1.5), whereas the scalar bottom masses of scenario $B$ are relatively light and the decay $\tilde{\chi}_{j}^{-} \rightarrow W^{-} \tilde{\chi}_{1}^{0}$ is not allowed. For the matter of simplicity, our numerical investigation is done for the first and second generation leptons where an influence of their Yukawa couplings can be safely neglected.

In figure [1] we show the CP asymmetries that are based on the triple products, (3.2)(3.4), in the decays $\tilde{b}_{1} \rightarrow t \tilde{\chi}_{1}^{-}, t \rightarrow b l^{+} \nu(b c \bar{s})$ and $\tilde{\chi}_{1}^{-} \rightarrow \ell_{i}^{-} \bar{\nu} \tilde{\chi}_{1}^{0}$ as a function of $\phi_{A_{b}}$. Figure 1(a) shows the three CP asymmetries $A_{\ell_{i}^{-} b t}$ that are based on the triple products in eq. (3.2) associated with the three different decay chains $\tilde{\chi}_{1}^{-} \rightarrow \ell_{1}^{-} \tilde{\tilde{\nu}} \rightarrow \ell_{1}^{-} \tilde{\nu} \tilde{\chi}_{1}^{0}$ (dashed line), $\tilde{\chi}_{1}^{-} \rightarrow \tilde{\ell}_{R}^{-} \nu \rightarrow \ell_{2}^{-} \bar{\nu} \tilde{\chi}_{1}^{0}$ (dotted line) and $\tilde{\chi}_{1}^{-} \rightarrow W^{-} \tilde{\chi}_{1}^{0} \rightarrow \ell_{3}^{-} \bar{\nu} \tilde{\chi}_{1}^{0}$ (dashdotdotted line). Figure 1 (a) also shows the CP asymmetry $A_{\ell^{-} \text {bt }}$ (solid line), eq. (3.5), where it is not necessary

| Scenario | A | B |
| :---: | :---: | :---: |
| $M_{2}$ | 350 | 250 |
| $\|\mu\|$ | 310 | 140 |
| $\phi_{\mu}=\phi_{M_{1}}$ | 0 | 0 |
| $\tan \beta$ | 3 | 3 |
| $\left\|A_{b}\right\|$ | 1200 | 1000 |
| $m_{\tilde{b}_{1}}$ | 480 | 320 |
| $m_{\tilde{b}_{2}}$ | 600 | 420 |
| $m_{\tilde{\ell}_{R}}$ | 200 | 100 |
| $m_{\tilde{\ell}_{L}}$ | 220 | 120 |
| $m_{\tilde{\nu}}$ | 208.1 | 96.4 |
| $m_{\tilde{\chi}_{1}^{0}}$ | 164.3 | 80.3 |
| $m_{\tilde{\chi}_{1}^{-}}$ | 257.3 | 107.7 |

Table 1: Input parameters $M_{2},|\mu|, \phi_{\mu}, \tan \beta,\left|A_{b}\right|, \phi_{A_{b}}, m_{\tilde{b}_{1}}, m_{\tilde{b}_{2}}, m_{\tilde{\ell}_{R}}$ and $m_{\tilde{\ell}_{L}}$ for scenarios A and B. All mass dimension parameters are given in GeV .
to distinguish the leptons from the different decay chains of the chargino. The asymmetry $A_{\ell_{1}^{-} b t}$ is the largest one with a maximum value of about $11 \%$. The CP asymmetries $A_{\ell_{2}^{-} b t}$ and $A_{\ell_{3}^{-} b t}$ have an additional phase space factor and are therefore suppressed compared to $A_{\ell_{1}^{-} b t}$.

We now estimate the number of scalar bottoms $\tilde{b}_{1}$ necessary to observe the CP asymmetries for a given number of standard deviations $\mathcal{N}_{\sigma}$ by

$$
\begin{equation*}
N_{\tilde{b}_{1}}=\frac{\mathcal{N}_{\sigma}{ }^{2}}{A_{i j k}^{2} B R\left(\tilde{b}_{1} \rightarrow t \tilde{\chi}_{1}^{-}\right)\left(\sum_{\ell=e, \mu} B R\left(\tilde{\chi}_{1}^{-} \rightarrow \ell_{i}^{-} \bar{\nu} \tilde{\chi}_{1}^{0}\right)\right)\left(\sum_{f} B R\left(W^{+} \rightarrow f\right)\right)}, \tag{4.2}
\end{equation*}
$$

where $f$ denotes the final state of the $W^{+}$decay considered, i.e. $f=u \bar{d}, c \bar{s}$, or $l^{+} \nu_{l}, l=e, \mu$. We calculate the branching ratios of $\tilde{b}_{1}$ using the formulae of the second paper in (9). For scenario A we obtain $B R\left(\tilde{b}_{1} \rightarrow t \tilde{\chi}_{1}^{-}\right)=4.9 \%$. Purely for the sake of simplicity, we calculate the chargino branching ratios $B R\left(\tilde{\chi}_{1}^{-} \rightarrow \ell_{i}^{-} \tilde{\nu}_{1}^{0}\right)$ assuming that scalar tau mixing can be neglected and that the lighter scalar leptons have a common mass $m_{\tilde{\ell}_{R}}$, the heavier scalar leptons have a common mass $m_{\tilde{\ell}_{L}}$ and the scalar neutrinos have a common mass given by

$$
\begin{equation*}
m_{\tilde{\nu}_{\ell}}=\sqrt{m_{\tilde{\ell}_{L}}^{2}+m_{Z}^{2} \cos 2 \beta \cos ^{2} \theta_{W}} . \tag{4.3}
\end{equation*}
$$

This means that the partial decay widths $\Gamma\left(\tilde{\chi}_{1}^{-} \rightarrow \ell^{-} \overline{\tilde{\nu}}_{\ell}\right)$ are equal for $\ell=e, \mu, \tau$. The same holds for the partial decay widths $\Gamma\left(\tilde{\chi}_{1}^{-} \rightarrow \tilde{\ell}_{R}^{-} \bar{\nu}_{\ell}\right)$ and $\Gamma\left(\tilde{\ell}_{R}^{-} \rightarrow \tilde{\chi}_{1}^{0} \ell^{-}\right)$. Then we obtain $\sum_{\ell=e, \mu} B R\left(\tilde{\chi}_{1}^{-} \rightarrow \ell_{i}^{-} \bar{\nu} \tilde{\chi}_{1}^{0}\right)=(31.3 \%, 30.7 \%, 1.5 \%)$ corresponding to the three different decay chains of $\tilde{\chi}_{1}^{-}$, (1.3)-(1.5). The values of the branching ratios of the $W$ boson are given by $B R\left(W^{+} \rightarrow \sum_{l} l^{+} \nu\right)=21.4 \%(l=e, \mu), B R\left(W^{+} \rightarrow \sum_{q} q \bar{q}^{\prime}=68 \%\right)$ and $B R\left(W^{+} \rightarrow c \bar{s}=32 \%\right)$ [ 9$]$. For an observation of the CP asymmetry $A_{\ell_{1}^{-} b t}$ at the 3- $\sigma$ level, at least $7.1 \cdot 10^{4}$ scalar bottoms have to be produced. The required number of scalar


Figure 1: CP asymmetries $A_{i j k}$ which are based on the triple products (a) $\left(\mathbf{p}_{\ell_{i}^{-}} \mathbf{p}_{b} \mathbf{p}_{t}\right)$, (b) $\left(\mathbf{p}_{\ell_{i}^{-}} \mathbf{p}_{l^{+}} \mathbf{p}_{b}\right)$, (c) ( $\left.\mathbf{p}_{\ell_{i}^{-}} \mathbf{p}_{c} \mathbf{p}_{t}\right)$ and (d) $\left(\mathbf{p}_{\ell_{i}^{-}} \mathbf{p}_{c} \mathbf{p}_{s}\right)$ for the decays $\tilde{b}_{1} \rightarrow t \tilde{\chi}_{1}^{-}, t \rightarrow b l^{+} \nu(b c \bar{s})$ and $\tilde{\chi}_{1}^{-} \rightarrow \ell_{i}^{-} \bar{\nu} \tilde{\chi}_{1}^{0}$, as a function of $\phi_{A_{b}}$. The lepton $\ell_{1}^{-}\left(\ell_{2}^{-}, \ell_{3}^{-}\right)$stems from the decay $\tilde{\chi}_{1}^{-} \rightarrow \ell_{1}^{-} \overline{\tilde{\nu}} \rightarrow \ell_{1}^{-} \bar{\nu} \tilde{\chi}_{1}^{0}$ $\left(\tilde{\chi}_{1}^{-} \rightarrow \tilde{\ell}_{R}^{-} \bar{\nu} \rightarrow \ell_{2}^{-} \bar{\nu} \tilde{\chi}_{1}^{0}, \tilde{\chi}_{1}^{-} \rightarrow W^{-} \tilde{\chi}_{1}^{0} \rightarrow \ell_{3}^{-} \tilde{\nu}_{1}^{0}\right)$. The corresponding asymmetries are shown as dashed lines (dotted lines, dashdotdotted lines). The solid lines represent the combined asymmetries in eq. (3.5). The MSSM parameters are for scenario A of table 1 .
bottoms in order to measure the asymmetry $A_{\ell-b t}=6.4 \%\left(\phi_{A_{b}}=0.5 \pi\right)$ at the $3-\sigma$ level is $1 \cdot 10^{5}$.

In figure 1(b) we plot the CP asymmetries that are based on the triple products $\left(\mathbf{p}_{\ell_{i}^{-}} \mathbf{p}_{l+}+\mathbf{p}_{b}\right)$, eq. (3.3), as a function of $\phi_{A_{b}}$. For the same reason as above the largest asymmetry is due to the chargino decay chain $\tilde{\chi}_{1}^{-} \rightarrow \ell_{1}^{-} \overline{\tilde{\nu}} \rightarrow \ell_{1}^{-} \bar{\nu} \tilde{\chi}_{1}^{0}$, (1.3). Its maximal value of about $13 \%$ is reached at $\phi_{A_{b}}=0.5 \pi$ and the number of scalar bottoms necessary to measure $A_{\ell_{1}^{-} l^{+} b}$ at the $3-\sigma$ level is about $1.5 \cdot 10^{5}$. Figure 1 (c) shows the CP asymmetries that are based on ( $\mathbf{p}_{\ell_{i}} \mathbf{p}_{c} \mathbf{p}_{t}$ ) as a function of $\phi_{A_{b}}$. The asymmetries shown in figure $]_{\text {(c) }}$ ) are more than twice as large as the asymmetries shown in figure 1. Their relative magnitudes can be attributed $(i)$ to the different sensitivity factors of the top quark polarization which is $\alpha_{l}=1$ for the asymmetries in figure 1 (b),(c),(d) and $\alpha_{b} \simeq 0.4$ for the asymmetries in figure (a), and (ii) to the different 3 -vectors involved in the triple products: for figures (a) and $\mathbb{1}(c)$ it is $\mathbf{p}_{t}$, while for figures $\mathbb{1}(b)$ and $\mathbb{1}(\mathrm{d})$ it is the 3 -vector of any of the decay products of the $t$-quark, which is always smaller or at most equal in magnitude than $\left|\mathbf{p}_{t}\right|$. For $\phi_{A_{b}}=0.5 \pi$ the CP asymmetry $A_{\ell_{1}^{-} c t}$ is about $27 \%$, which means that $2.5 \cdot 10^{4}$ scalar


Figure 2: CP asymmetries $A_{i j k}$ that are based on the triple products ( $\mathbf{p}_{W^{-}} \mathbf{p}_{c} \mathbf{p}_{t}$ ) (solid line), $\left(\mathbf{p}_{W-} \mathbf{p}_{c} \mathbf{p}_{\bar{s}}\right)$ (dotted line), $\left(\mathbf{p}_{W-} \mathbf{p}_{b} \mathbf{p}_{t}\right)$ (dashed line) and $\left(\mathbf{p}_{W-} \mathbf{p}_{l+} \mathbf{p}_{b}\right)$ (dashdotdotted line) for the process $\tilde{b}_{1} \rightarrow t \tilde{\chi}_{1}^{-}, t \rightarrow b l^{+} \nu(b c \bar{s})$ and $\tilde{\chi}_{1}^{-} \rightarrow W^{-} \tilde{\chi}_{1}^{0} \rightarrow \bar{c} s \tilde{\chi}_{1}^{0}$ as a function of $\phi_{A_{b}}$. The MSSM parameters are given in table 1 (scenario A ).
bottoms are necessary for its measurement at $3-\sigma$. The combined asymmetry in eq. (3.5) can be as large as about $16 \%$ and the appropriate number of scalar bottoms to probe it at the $3-\sigma$ level is $3.6 \cdot 10^{4}$. In figure 1(d) we plot the CP asymmetries which are based on the triple products $\left(\mathbf{p}_{\ell_{i}^{-}} \mathbf{p}_{c} \mathbf{p}_{\bar{s}}\right)$. For $\phi_{A_{b}}=0.5 \pi$ the asymmetry is $A_{\ell_{1}^{-} c \bar{s}}$ of about $10 \%$ and at least $1.9 \cdot 10^{5}$ scalar bottoms are required for its measurement.

In figure 2 we show the CP asymmetries that are based on the triple products $\left(\mathbf{p}_{W^{-}} \mathbf{p}_{c} \mathbf{p}_{t}\right),\left(\mathbf{p}_{W^{-}} \mathbf{p}_{c} \mathbf{p}_{s}\right),\left(\mathbf{p}_{W^{-}} \mathbf{p}_{b} \mathbf{p}_{t}\right)$ and $\left(\mathbf{p}_{W^{-}} \mathbf{p}_{l^{+}} \mathbf{p}_{b}\right)$ as a function of $\phi_{A_{b}}$ for scenario A given in table 1. The momentum vector $\mathbf{p}_{W^{-}}$involved in the triple products is that of the $W$ boson stemming from the decay $\tilde{\chi}_{1}^{-} \rightarrow W^{-} \tilde{\chi}_{1}^{0}$. The largest asymmetry $A_{W^{-} c t}$ attains its maximum value of about $6 \%$ at $\phi_{A_{b}}=0.5 \pi$. For the theoretical estimate of the number of scalar bottoms necessary to observe this asymmetry we replace $\sum_{\ell=e, \mu} B R\left(\tilde{\chi}_{1}^{-} \rightarrow \ell_{i}^{-} \bar{\nu} \tilde{\chi}_{1}^{0}\right)$ by $B R\left(\tilde{\chi}_{1}^{-} \rightarrow W^{-} \tilde{\chi}_{1}^{0}\right) \cdot \sum_{q} B R\left(W^{-} \rightarrow \bar{q} q^{\prime}\right)=4.8 \%$ in eq. (4.2). We then obtain that $1.1 \cdot 10^{6}$ scalar bottoms are required for a $3-\sigma$ evidence.

In figure 3 the CP asymmetries for scenario B of table 11 are displayed. In this case the decay $\tilde{\chi}_{1}^{-} \rightarrow W^{-} \tilde{\chi}_{1}^{0}$ is kinematically not accessible. We plot the CP asymmetries for the decay chains $\tilde{\chi}_{1}^{-} \rightarrow \ell_{1}^{-} \overline{\tilde{\nu}} \rightarrow \ell_{1}^{-} \bar{\nu} \tilde{\chi}_{1}^{0}$ and $\tilde{\chi}_{1}^{-} \rightarrow \tilde{\ell}_{R}^{-} \bar{\nu} \rightarrow \ell_{2}^{-} \bar{\nu} \tilde{\chi}_{1}^{0}$ as a function of $\phi_{A_{b}}$. For the branching ratios we obtain $B R\left(\tilde{b}_{1} \rightarrow t \tilde{\chi}_{1}^{-}\right)=7.2 \%, \sum_{\ell=e, \mu} B R\left(\tilde{\chi}_{1}^{-} \rightarrow \ell_{1}^{-} \bar{\nu} \tilde{\chi}_{1}^{0}\right)=54.3 \%$ and $\sum_{\ell=e, \mu} B R\left(\tilde{\chi}_{1}^{-} \rightarrow \ell_{2}^{-} \bar{\nu} \tilde{\chi}_{1}^{0}\right)=12.4 \%$ in scenario B. Figure $3($ a) shows the CP asymmetries which are based on the triple products given in eq. (3.2). The largest asymmetry results from the triple product $\left(\mathbf{p}_{\ell_{1}^{-}} \mathbf{p}_{b} \mathbf{p}_{t}\right)$ where the lepton $\ell_{1}^{-}$originates from the first step of the decay chain $\tilde{\chi}_{1}^{-} \rightarrow \ell_{1}^{-} \overline{\tilde{\nu}}^{1} \rightarrow \ell_{1}^{-} \bar{\nu} \tilde{\chi}_{1}^{0}$, and its maximum value is of about $15 \%$. For its measurement (at $3-\sigma$ ) $16 \cdot 10^{4}$ scalar bottoms are required. The CP asymmetry that is based on $\left(\mathbf{p}_{\ell_{2}^{-}} \mathbf{p}_{b} \mathbf{p}_{t}\right)$, where the lepton $\ell_{2}^{-}$comes from the decay chain $\tilde{\chi}_{1}^{-} \rightarrow \tilde{\ell}_{R}^{-} \bar{\nu} \rightarrow \ell_{2}^{-} \bar{\nu} \tilde{\chi}_{1}^{0}$


Figure 3: CP asymmetries $A_{i j k}$ that are based on the triple products (a) $\left(\mathbf{p}_{\ell_{i}^{-}} \mathbf{p}_{b} \mathbf{p}_{t}\right)$, (b) $\left(\mathbf{p}_{\ell_{i}^{-}} \mathbf{p}_{l^{+}} \mathbf{p}_{b}\right)$, (c) $\left(\mathbf{p}_{\ell_{i}^{-}} \mathbf{p}_{c} \mathbf{p}_{t}\right)$ and (d) $\left(\mathbf{p}_{\ell_{i}^{-}} \mathbf{p}_{c} \mathbf{p}_{s}\right)$ for the decays $\tilde{b}_{1} \rightarrow t \tilde{\chi}_{1}^{-}, t \rightarrow b l^{+} \nu(b c \bar{s})$ and $\tilde{\chi}_{1}^{-} \rightarrow \ell_{i}^{-} \bar{\nu} \tilde{\chi}_{1}^{0}$, as a function of $\phi_{A_{b}}$. The lepton $\ell_{1}^{-}\left(\ell_{2}^{-}\right)$stems from the decay $\tilde{\chi}_{1}^{-} \rightarrow \ell_{1}^{-} \overline{\tilde{\nu}} \rightarrow \ell_{1}^{-} \bar{\nu} \tilde{\chi}_{1}^{0}$ $\left(\tilde{\chi}_{1}^{-} \rightarrow \tilde{\ell}_{R}^{-} \bar{\nu} \rightarrow \ell_{2}^{-} \bar{\nu}_{1}^{0}\right)$. The corresponding asymmetries are shown as dashed lines (dotted lines). The solid lines represent the combined asymmetries in eq. (3.5). The MSSM parameters are for scenario B of table 1.
is phase space suppressed. Due to the large branching ratio of $\tilde{\chi}_{1}^{-} \rightarrow \ell_{1}^{-} \bar{\nu} \tilde{\chi}_{1}^{0}$ the combined asymmetry, eq. (3.5), is about $12 \%$, therefore $1.9 \cdot 10^{4}$ scalar bottoms would be necessary for a measurement at the $3-\sigma$ level. In figure $3(\mathrm{~b})$ we plot the CP asymmetries that are based on the triple products defined in eq. (3.3). The largest asymmetry $A_{\ell_{1}^{-} l^{+} b}$ reaches its maximum value of about $13 \%$ at $\phi_{A_{b}}=1.5 \pi$. Figure 回(c) shows the CP asymmetry formed with the triple products $\left(\mathbf{p}_{\ell_{i}^{-}} \mathbf{p}_{c} \mathbf{p}_{t}\right)$. As expected, the asymmetry $A_{\ell_{1}^{-} c t}$ is the largest and its maximum value is of about $36 \%$. In this case $5.5 \cdot 10^{3}$ scalar bottoms are necessary for a measurement of $A_{\ell_{1}^{-} c t}$ at the $3-\sigma$ level. The CP asymmetry, where it is not necessary to distinguish from which $\tilde{\chi}_{1}^{-}$decay chain the lepton originates, reaches a maximum of about $30 \%$. In this case the production of $6.7 \cdot 10^{3}$ scalar bottoms is necessary to probe the asymmetry $A_{\ell^{-} c t}$ at $3-\sigma$. In figure $3(\mathrm{~d})$ the CP asymmetries that are based on the triple products $\left(\mathbf{p}_{\ell_{i}^{-}} \mathbf{p}_{c} \mathbf{p}_{s}\right)$ are displayed. The maximum of $A_{\ell_{1}^{-} c s}$ is about $9 \%$, which means that 1.1. $10^{5}$ scalar bottoms are necessary to determine (at $3-\sigma$ ) that the asymmetry is non-zero.
 plane. The slepton masses are $m_{\tilde{\ell}_{L}}=140 \mathrm{GeV}$ and $m_{\tilde{\ell}_{R}}=110 \mathrm{GeV}, \tan \beta=10$ and the


Figure 4: Contours in the $\phi_{\mu}-\phi_{A_{b}}$ plane of the combined asymmetry, eq. (3.5), which is based on $\left(\mathbf{p}_{\ell-} \mathbf{p}_{c} \mathbf{p}_{t}\right)$. We take $m_{\tilde{\mathscr{R}}_{L}}=140 \mathrm{GeV}, m_{\tilde{\ell}_{R}}=110 \mathrm{GeV}$ and $\tan \beta=10$, the other parameters are as in scenario $B$ of table 1 .
other parameters are as given as in scenario B of table 1. For the scenario chosen, the first term of eq. (4.1) is small compared to the second term because $\cos \theta_{\tilde{b}} \ll \sin \theta_{\tilde{b}}$. Hence, the behavior of the asymmetry is given by the second term of eq. (4.1), which is small in the $\phi_{\mu}-\phi_{A_{b}}$ plane where $\phi_{\mu}+\phi_{A_{b}} \approx 0, \pi$ because there $\phi_{\tilde{b}}-\arg \left[U_{12}^{*} V_{12}^{*}\right] \approx 0, \pi$ resulting in a cancellation of the two terms in eq. (4.1). For CP phases of $\phi_{\mu} \approx 0.8 \pi$ and $\phi_{A_{b}} \approx 0.6 \pi$ the asymmetry reaches its maximum of about $11 \%$.

## 5. Summary

We have proposed various T-odd asymmetries in the decay $\tilde{b}_{m} \rightarrow t \tilde{\chi}_{j}^{-}$, which are based on triple product correlations that involve the polarization vectors of $t$ and $\tilde{\chi}_{j}^{-}$. The distributions of their decay products depend on the polarizations of $t$ and $\tilde{\chi}_{j}^{-}$. For the $\tilde{\chi}_{j}^{-}$decay into a leptonic final state $\ell^{-} \bar{\nu} \tilde{\chi}_{1}^{0}$ we have considered the three possible decay chains $\tilde{\chi}_{j}^{-} \rightarrow \ell^{-} \overline{\tilde{\nu}} \rightarrow \ell^{-} \tilde{\nu}_{1}^{0}, \tilde{\chi}_{j}^{-} \rightarrow \tilde{\ell}_{n}^{-} \rightarrow \ell^{-} \bar{\nu} \tilde{\chi}_{1}^{0}$ and $\tilde{\chi}_{j}^{-} \rightarrow W^{-} \tilde{\chi}_{1}^{0} \rightarrow \ell^{-} \bar{\nu} \tilde{\chi}_{1}^{0}$. We have also considered the 2-body decay $\tilde{\chi}_{j}^{-} \rightarrow W^{-} \chi_{1}^{0}$, where the $W$ boson decays hadronically. The proposed T-odd asymmetries are proportional to the product of left- and right-couplings $t \tilde{b}_{m} \tilde{\chi}_{k}^{-}$and are non-vanishing due to non-zero phases $\phi_{\mu}$ and/or $\phi_{A_{b}}$. Since scalar bottom mixing can be large these asymmetries will allow us to determine the CP violating phase $\phi_{A_{b}}$, which is not easily accessible otherwise. We have also pointed out that true CP violating asymmetries can be obtained by summing the T-odd asymmetries that arise in the decays $\tilde{b}_{m} \rightarrow \tilde{\chi}_{j}^{-} t$ and $\overline{\tilde{b}}_{m} \rightarrow \tilde{\chi}_{j}^{+} \bar{t}$. In this case an identification of the charges of the involved particles is not necessary.

In a numerical study we have presented results of these asymmetries for the decay $\tilde{b}_{1} \rightarrow t \tilde{\chi}_{1}^{-}$. The asymmetry $A_{\ell_{1}^{-} c t}$, which is based on the triple product $\left(\mathbf{p}_{\ell_{1}^{-}} \mathbf{p}_{c} \mathbf{p}_{t}\right)$, is the
largest one and its magnitude can be of the order $40 \%$. We have also defined the asymmetry $A_{\ell^{-} c t}$, eq. (3.5), which is based on $\left(\mathbf{p}_{\ell^{-}} \mathbf{p}_{c} \mathbf{p}_{t}\right)$, and where it is not necessary to distinguish between the different leptonic $\tilde{\chi}_{1}^{-}$decay chains. We have found that this asymmetry can go up to $30 \%$. By making a theoretical estimate of the number of $\tilde{b}_{1}$ necessary to observe the T-odd asymmetries we have found that a $\tilde{b}_{1}$ production rate of $O\left(10^{3}\right)$ will be necessary to observe some of the proposed asymmetries, which should be possible at the LHC or at a future linear collider.

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## A. Scalar bottom masses and mixing

The left-right mixing of the scalar bottoms is described by a hermitian $2 \times 2$ mass matrix which in the basis $\left(\tilde{b}_{L}, \tilde{b}_{R}\right)$ reads
where

$$
\begin{align*}
& M_{\tilde{b}_{L L}}^{2}=M_{\tilde{Q}}^{2}+\left(-\frac{1}{2}+\frac{1}{3} \sin ^{2} \Theta_{W}\right) \cos 2 \beta m_{Z}^{2}+m_{b}^{2}  \tag{A.2}\\
& M_{\tilde{b}_{R R}}^{2}=M_{\tilde{D}}^{2}-\frac{1}{3} \sin ^{2} \Theta_{W} \cos 2 \beta m_{Z}^{2}+m_{b}^{2}  \tag{A.3}\\
& M_{\tilde{b}_{R L}}^{2}=\left(M_{\tilde{b}_{L R}}^{2}\right)^{*}=m_{b}\left(A_{b}-\mu^{*} \tan \beta\right)  \tag{A.4}\\
& \quad \phi_{\tilde{b}}=\arg \left[A_{b}-\mu^{*} \tan \beta\right] \tag{A.5}
\end{align*}
$$

where $\tan \beta=v_{2} / v_{1}$ with $v_{1}\left(v_{2}\right)$ being the vacuum expectation value of the Higgs field $H_{1}^{0}\left(H_{2}^{0}\right), m_{b}$ is the mass of the bottom quark and $\Theta_{W}$ is the weak mixing angle, $\mu$ is the Higgs-higgsino mass parameter and $M_{\tilde{Q}}, M_{\tilde{D}}, A_{t}$ are the soft SUSY-breaking parameters of the scalar bottom system. The mass eigenstates $\tilde{b}_{i}$ are $\left(\tilde{b}_{1}, \tilde{b}_{2}\right)=\left(\tilde{b}_{L}, \tilde{b}_{R}\right) \mathcal{R}^{\tilde{b}^{T}}$ with

$$
\mathcal{R}^{\tilde{b}}=\left(\begin{array}{cc}
e^{i \phi_{\tilde{b}} \cos \theta_{\tilde{b}}} & \sin \theta_{\tilde{b}}  \tag{A.6}\\
-\sin \theta_{\tilde{b}} & e^{-i \phi_{\tilde{b}}} \cos \theta_{\tilde{b}}
\end{array}\right)
$$

with

$$
\begin{equation*}
\cos \theta_{\tilde{b}}=\frac{-\left|M_{\tilde{b}_{L R}}^{2}\right|}{\sqrt{\left|M_{\tilde{b}_{L R}}^{2}\right|^{2}+\left(m_{\tilde{b}_{1}}^{2}-M_{\tilde{b}_{L L}}^{2}\right)^{2}}}, \quad \sin \theta_{\tilde{b}}=\frac{M_{\tilde{b}_{L L}}^{2}-m_{\tilde{b}_{1}}^{2}}{\sqrt{\left|M_{\tilde{b}_{L R}}^{2}\right|^{2}+\left(m_{\tilde{b}_{1}}^{2}-M_{\tilde{b}_{L L}}^{2}\right)^{2}}} . \tag{A.7}
\end{equation*}
$$

The mass eigenvalues are

$$
\begin{equation*}
m_{\tilde{b}_{1,2}}^{2}=\frac{1}{2}\left(\left(M_{\tilde{b}_{L L}}^{2}+M_{\tilde{b}_{R R}}^{2}\right) \mp \sqrt{\left(M_{\tilde{b}_{L L}}^{2}-M_{\tilde{b}_{R R}}^{2}\right)^{2}+4\left|M_{\tilde{b}_{L R}}^{2}\right|^{2}}\right) . \tag{A.8}
\end{equation*}
$$

## B. Lagrangian and couplings

The parts of the Lagrangian, necessary to calculate the decay rates of $\tilde{b}_{m} \rightarrow \tilde{\chi}_{j}^{-} t$ with the subsequent decays $\tilde{\chi}_{j}^{-} \rightarrow \ell^{-} \bar{\nu} \tilde{\chi}_{1}^{0}$ are

$$
\begin{align*}
\mathcal{L}_{t \tilde{b} \chi^{+}} & =g \bar{t}\left(l_{m j}^{\tilde{b}} P_{R}+k_{m j}^{\tilde{b}} P_{L}\right) \tilde{\chi}_{j}^{+} \tilde{b}_{m}+\text { h.c. },  \tag{B.1}\\
\mathcal{L}_{\ell \tilde{\nu} \tilde{\chi}^{+}} & =g \bar{\ell}\left(k_{j}^{\tilde{\nu}} P_{L}+l_{j}^{\tilde{\nu}} P_{R}\right) \tilde{\chi}_{j}^{+C} \tilde{\nu}_{\ell}+\text { h.c. },  \tag{B.2}\\
\mathcal{L}_{\nu \tilde{\ell} \tilde{\chi}^{+}} & =g l_{n j}^{\tilde{\ell}} \overline{\nu_{\ell}} P_{R} \tilde{\chi}_{j}^{+} \tilde{\ell}_{n}+\text { h.c. },  \tag{B.3}\\
\mathcal{L}_{W-\tilde{\chi}^{+} \tilde{\chi}^{0}} & =g W_{\mu}^{-} \overline{\tilde{\chi}_{k}^{0}} \gamma^{\mu}\left(O_{k j}^{L} P_{L}+O_{k j}^{R} P_{R}\right) \tilde{\chi}_{j}^{+}+\text {h.c. },  \tag{B.4}\\
\mathcal{L}_{\ell \tilde{\ell} \tilde{\chi}^{0}} & =g \bar{\ell}\left(a_{n k}^{\tilde{\ell}} P_{R}+b_{n k}^{\tilde{\ell}} P_{L}\right) \tilde{\chi}_{k}^{0} \tilde{\ell}_{n}+\text { h.c. },  \tag{B.5}\\
\mathcal{L}_{\nu \tilde{\nu} \tilde{\chi}^{0}} & =g f_{L k}^{\nu} \overline{\bar{l}_{\ell}} P_{R} \tilde{\chi}_{k}^{0} \tilde{\nu}_{\ell}+\text { h.c. }, \tag{B.6}
\end{align*}
$$

where the couplings are defined as

$$
\begin{gather*}
l_{m j}^{\tilde{b}}=-\mathcal{R}_{m 1}^{\tilde{b} *} U_{j 1}+Y_{b} \mathcal{R}_{m 2}^{\tilde{b} *} U_{j 2}, \quad k_{m j}^{\tilde{b}}=\mathcal{R}_{m 1}^{\tilde{b} *} Y_{t} V_{j 2}^{*},  \tag{B.7}\\
l_{j}^{\tilde{\tilde{j}}}=-V_{j 1}, \quad k_{j}^{\tilde{\nu}}=Y_{\ell} U_{j 2}^{*},  \tag{B.8}\\
a_{n k}^{\tilde{\ell}}=\mathcal{R}_{n 1}^{\tilde{\ell} *} f_{L k}^{\ell}+\mathcal{R}_{n 2}^{\tilde{\ell} *} h_{R k}^{\ell}, \quad b_{n k}^{\tilde{\ell}}=\mathcal{R}_{n 1}^{\tilde{\mathcal{E} *}} h_{L k}^{\ell}+\mathcal{R}_{n 2}^{\tilde{\ell} *} f_{R k}^{\ell},  \tag{B.9}\\
f_{L k}^{\ell}=\frac{1}{\sqrt{2}}\left(N_{k 2}+\tan \theta_{W} N_{k 1}\right), \\
f_{R k}^{\ell}=-\sqrt{2} \tan \theta_{W} N_{k 1}^{*}, \\
h_{R k}^{\ell}=\left(h_{L k}^{l}\right)^{*}=-Y_{\ell} N_{k 3}, \\
f_{L k}^{\nu}=\frac{1}{\sqrt{2}}\left(\tan \theta_{W} N_{k 1}-N_{k 2}\right),  \tag{B.10}\\
l_{n j}^{\tilde{l}}=-\mathcal{R}_{n 1}^{\tilde{\ell} *} U_{j 1}+Y_{\ell} \mathcal{R}_{n 2}^{\tilde{\ell} *} U_{j 2},  \tag{B.11}\\
O_{k j}^{L}=-\frac{1}{\sqrt{2}} N_{k 4} V_{j 2}^{*}+N_{k 2} V_{j 1}^{*}, \quad O_{k j}^{R}=\frac{1}{\sqrt{2}} N_{k 3}^{*} U_{j 2}+N_{k 2}^{*} U_{j 1}, \tag{B.12}
\end{gather*}
$$

where in the above equations $U$ and $V$ are the unitary $2 \times 2$ mixing matrices that diagonalize the chargino mass matrix $\mathcal{M}_{C}, U^{*} \mathcal{M}_{C} V^{-1}=\operatorname{diag}\left(m_{\chi_{1}}, m_{\chi_{2}}\right), N_{i j}$ is the complex unitary $4 \times 4$ matrix which diagonalizes the neutral gaugino-higgsino mass matrix $Y_{\alpha \beta}, N_{i \alpha}^{*} Y_{\alpha \beta} N_{k \beta}^{*}=m_{\chi_{i}^{0}} \delta_{i k}$, in the basis $\left(\tilde{B}, \tilde{W}^{3}, \tilde{H}_{1}^{0}, \tilde{H}_{2}^{0}\right)$ [2], $\mathcal{R}^{\tilde{\ell}}$ is the mixing matrix in the slepton sector (see for instance [8]) and the Yukawa couplings are given by $Y_{t}=m_{t} /\left(\sqrt{2} m_{W} \sin \beta\right), Y_{b}=m_{b} /\left(\sqrt{2} m_{W} \cos \beta\right)$ and $Y_{\ell}=m_{\ell} /\left(\sqrt{2} m_{W} \cos \beta\right)$, with $m_{W}$ being the mass of the $W$ boson.

## C. Phase space and kinematics

We will work in the rest frame of $\tilde{b}_{m}$ and we fix the coordinate system so that the chargino momentum $\mathbf{p}_{\chi_{j}}$ points along the $Z$-axis.

Phase space element of the decay $\tilde{b}_{m} \rightarrow \tilde{\chi}_{j}^{-} t$ :

$$
\begin{equation*}
d \Phi_{\tilde{b}_{m}}=\frac{\left|\mathbf{p}_{t}\right|}{4 \pi m_{\tilde{b}_{m}}}, \quad\left|\mathbf{p}_{t}\right|=\frac{\lambda^{\frac{1}{2}}\left(m_{\tilde{b}_{m}}^{2}, m_{t}^{2}, m_{\chi_{j}}^{2}\right)}{2 m_{\tilde{b}_{m}}} \tag{C.1}
\end{equation*}
$$

where $\lambda(x, y, z)=x^{2}+y^{2}+z^{2}-2(x y+x z+y z)$.
Phase space elements of the top decays (1.2):
The phase space element of the top decay $t \rightarrow b W^{+}$is given as

$$
\begin{equation*}
d \Phi_{t}^{b}=\frac{E_{b}^{2}}{2\left(m_{t}^{2}-m_{W}^{2}\right)} \frac{d \Omega_{b}}{(2 \pi)^{2}}, \quad E_{b}=\frac{m_{t}^{2}-m_{W}^{2}}{2\left(E_{t}+\left|\mathbf{p}_{t}\right| c_{b}\right)} . \tag{C.2}
\end{equation*}
$$

The phase space element of the top decay $t \rightarrow b l^{+} \nu$ reads

$$
\begin{equation*}
d \Phi_{t}^{l}=\frac{1}{2 \pi} d \Phi_{t}^{b} d \Phi_{W} \tag{C.3}
\end{equation*}
$$

where we used the narrow width approximation for the $W$ boson propagator. $d \Phi_{W}$ is the phase space element for $W^{+} \rightarrow l^{+} \nu_{l}$ :

$$
\begin{equation*}
d \Phi_{W}=\frac{E_{l}^{2}}{2 m_{W}^{2}} \frac{d \Omega_{l}}{(2 \pi)^{2}}, \quad E_{l}=\frac{m_{W}^{2}}{2\left[E_{t}+\left|\mathbf{p}_{t}\right| c_{l}-E_{b}\left(1-c_{b l}\right)\right]}, \tag{C.4}
\end{equation*}
$$

where $c_{b}=\cos \theta_{b}, c_{l}=\cos \theta_{l}$ and $c_{b l}=\cos \theta_{b l}$, with $\theta_{b l}$ being the angle between $\mathbf{p}_{b}$ and $\mathbf{p}_{l}$, and $d \Omega_{b}=\sin \theta_{b} d \theta_{b} d \phi_{b}$ etc.
Phase space element for $\tilde{\chi}_{j}^{-}$decay via $\tilde{\nu}$ exchange (1.3):
The phase space element of the decay $\tilde{\chi}_{j}^{-} \rightarrow \ell_{1}^{-} \overline{\tilde{\nu}}$ reads

$$
\begin{equation*}
d \Phi_{\chi_{j}}^{1}=\frac{E_{\ell_{1}}^{2}}{2\left(m_{\chi_{j}}^{2}-m_{\tilde{\nu}}^{2}\right)} \frac{d \Omega_{\ell_{1}}}{(2 \pi)^{2}}, \quad E_{\ell_{1}}=\frac{m_{\chi_{j}}^{2}-m_{\tilde{\tilde{}}}^{2}}{2\left(E_{\chi_{j}}-\left|\mathbf{p}_{\chi_{j}}\right| c_{1}\right)}, \tag{C.5}
\end{equation*}
$$

where $c_{1}=\cos \theta_{\ell_{1}}$.
Phase space elements for $\tilde{\chi}_{j}^{-}$decay via $\tilde{\ell}$ exchange (1.4):
The phase space element of the decay $\tilde{\chi}_{j}^{-} \rightarrow \tilde{\ell}_{n}^{-} \bar{\nu}$ is given by

$$
\begin{equation*}
d \Phi_{\chi_{j}}^{2}=\frac{E_{\nu}^{2}}{2\left(m_{\chi_{j}}^{2}-m_{\tilde{\ell}}^{2}\right)} \frac{d \Omega_{\nu}}{(2 \pi)^{2}}, \quad E_{\nu}=\frac{m_{\chi_{j}}^{2}-m_{\tilde{\ell}}^{2}}{2\left(E_{\chi_{j}}-\left|\mathbf{p}_{\chi_{j}}\right| c_{\nu}\right)}, \tag{C.6}
\end{equation*}
$$

where $c_{\nu}=\cos \theta_{\nu}$. For the subsequent decay $\tilde{\ell}_{n}^{-} \rightarrow \tilde{\chi}_{1}^{0} \ell_{2}^{-}$the phase space element reads

$$
\begin{equation*}
d \Phi_{\tilde{\ell}}=\frac{E_{\ell_{2}}^{2}}{2\left(m_{\tilde{\ell}}^{2}-m_{\chi_{1}^{0}}^{2}\right.} \frac{d \Omega_{\ell_{2}}}{(2 \pi)^{2}}, \quad E_{\ell_{2}}=\frac{m_{\tilde{\ell}}^{2}-m_{\chi_{1}^{0}}^{2}}{2\left(E_{\tilde{\ell}}-\left|\mathbf{p}_{\tilde{\ell}}\right| c_{\tilde{\ell} \ell_{2}}\right)}, \tag{C.7}
\end{equation*}
$$

where $c_{\tilde{\ell} \ell_{2}}=\cos \theta_{\tilde{\ell} \ell_{2}}$ being the angle between $\mathbf{p}_{\tilde{\ell}}$ and $\mathbf{p}_{\ell_{2}}$.
Phase space elements for $\tilde{\chi}_{j}^{-}$decay via $W$ boson exchange (1.5):
The phase space element of the decay $\tilde{\chi}_{j}^{-} \rightarrow W^{-} \tilde{\chi}_{1}^{0}$ is given by

$$
\begin{equation*}
\left(d \Phi_{\chi_{j}}^{3}\right)^{ \pm}=\frac{\left|\mathbf{p}_{W}^{ \pm}\right|^{2}}{4\left|E_{W}^{ \pm}\right| \mathbf{p}_{\chi_{j}}\left|\cos \theta_{W}-E_{\chi_{j}}\right| \mathbf{p}_{W}^{ \pm}| |} \frac{d \Omega_{W}}{(2 \pi)^{2}} \tag{C.8}
\end{equation*}
$$

with

$$
\begin{align*}
\left|\mathbf{p}_{W}^{ \pm}\right| & =\left[\left(m_{\chi_{j}}^{2}+m_{W}^{2}-m_{\chi_{1}^{0}}^{2}\right)\left|\mathbf{p}_{\chi_{j}}\right| \cos \theta_{W}\right) \\
& \left. \pm E_{\chi_{j}} \sqrt{\lambda\left(m_{\chi_{j}}^{2}, m_{W}^{2}, m_{\chi_{1}^{0}}^{2}\right)-4\left|\mathbf{p}_{\chi_{j}}\right|^{2} m_{W}^{2}\left(1-\cos ^{2} \theta_{W}\right)}\right] \\
& \left.\times\left[2\left|\mathbf{p}_{\chi_{j}}\right|^{2}\left(1-\cos ^{2} \theta_{W}\right)+2 m_{\chi_{j}}^{2}\right)\right]^{-1} . \tag{C.9}
\end{align*}
$$

There are two solutions $\left|\mathbf{p}_{W}^{ \pm}\right|$in the case $\left|\mathbf{p}_{\chi_{j}}^{0}\right|<\left|\mathbf{p}_{\chi_{j}}\right|$, where $\left|\mathbf{p}_{\chi_{j}}^{0}\right|=\frac{\sqrt{\lambda\left(m_{\chi_{j}}^{2}, m_{W}^{2}, m_{\chi_{1}^{0}}^{2}\right)}}{2 m_{W}}$ is the chargino momentum if the $W$ boson is produced at rest. The $W$ decay angle $\theta_{W}$ is constrained in that case and the maximal angle $\theta_{W}^{\max }$ is given as

$$
\begin{equation*}
\sin \theta_{W}^{\max }=\frac{\left|\mathbf{p}_{\chi_{j}}^{0}\right|}{\left|\mathbf{p}_{\chi_{j}}\right|}=\frac{m_{\tilde{b}_{m}}}{m_{W}} \frac{\lambda^{\frac{1}{2}}\left(m_{\chi_{j}}^{2}, m_{W}^{2}, m_{\chi_{1}^{0}}^{2}\right)}{\lambda^{\frac{1}{2}}\left(m_{\tilde{b}}^{2}, m_{\chi_{j}}^{2}, m_{t}^{2}\right)} \leq 1 \tag{C.10}
\end{equation*}
$$

If $\left|\mathbf{p}_{\chi_{j}}^{0}\right|>\left|\mathbf{p}_{\chi_{j}}\right|$, the decay angle $\theta_{W}$ is not constrained and there is only the physical solution $\left|\mathbf{p}_{W}^{+}\right|$.

For the subsequent decay of the $W$ boson, $W^{-} \rightarrow \ell_{3}^{-} \nu$, the phase space element is analogous to the one given in (C.4) and reads

$$
\begin{equation*}
d \Phi_{W}^{3}=\frac{E_{\ell_{3}}^{2}}{2 m_{W}^{2}} \frac{d \Omega_{\ell_{3}}}{(2 \pi)^{2}}, \quad E_{\ell_{3}}=\frac{m_{W}^{2}}{2\left(E_{W}^{ \pm}-\left|\mathbf{p}_{W}^{ \pm}\right| c_{\ell_{3} W}\right)} \tag{C.11}
\end{equation*}
$$

where $c_{\ell_{3} W}=\cos \theta_{\ell_{3} W}$ being the angle between $\mathbf{p}_{\ell_{3}}$ and $\mathbf{p}_{W}$.

## References

[1] H.P. Nilles, Supersymmetry, supergravity and particle physics, Phys. Rept. 110 (1984) 1; H.E. Haber and G.L. Kane, The search for supersymmetry: probing physics beyond the standard model, Phys. Rept. 117 (1985) 75;
R. Barbieri, Riv. Nuovo Cim. 11 (1988) 1.
[2] J.F. Gunion and H.E. Haber, Higgs bosons in supersymmetric models. 1, Nucl. Phys. B 272 (1986) 1 Erratum Nucl. Phys. B 402 (1993) 57; Higgs bosons in supersymmetric models. 2. implications for phenomenology, Nucl. Phys. B 278 (1986) 449.
[3] T. Ibrahim and P. Nath, The chromoelectric and purely gluonic operator contributions to the neutron electric dipole moment in $N=1$ supergravity, Phys. Lett. B 418 (1998) 98 hep-ph/9707409; The Neutron and the electron electric dipole moment in N=1 supergravity unification, Phys. Rev. D 57 (1998) 478, Erratum Phys. Rev. D 58 (1998) 019901, Erratum Phys. Rev. D 60 (1999) 079903 hep-ph/9708456; Phys. Rev. D 60 (1999) 119901; The neutron and the lepton edms in MSSM, large CP-violating phases and the cancellation mechanism, Phys. Rev. D 58 (1998) 111301 hep-ph/9807501; Large CP phases and the cancellation mechanism in edms in SUSY, string and brane models, Phys. Rev. D 61 (2000) 093004 hep-ph/9910553;
M. Brhlik, G.J. Good and G.L. Kane, Electric dipole moments do not require the CP-violating phases of supersymmetry to be small, Phys. Rev. D 59 (1999) 115004 hep-ph/9810457;
M. Brhlik, L.L. Everett, G.L. Kane and J.D. Lykken, A resolution to the supersymmetric $C P$ problem with large soft phases via D-branes, Phys. Rev. Lett. 83 (1999) 2124
hep-ph/9905215;
Superstring theory and CP-violating phases: can they be related?, Phys. Rev. D 62 (2000) 035005 hep-ph/9908326;
A. Bartl, T. Gajdosik, W. Porod, P. Stockinger and H. Stremnitzer, Electron and neutron electric dipole moments in the constrained MSSM, Phys. Rev. D 60 (1999) 073003
hep-ph/9903402;
V.D. Barger et al., CP-violating phases in SUSY, electric dipole moments and linear colliders, Phys. Rev. D 64 (2001) 056007 hep-ph/0101106;
A. Bartl et al., General flavor blind MSSM and CP-violation, Phys. Rev. D 64 (2001) 076009 hep-ph/0103324;
S. Abel, S. Khalil and O. Lebedev, Edm constraints in supersymmetric theories, Nucl. Phys. B 606 (2001) 151 hep-ph/0103320.
[4] A. Bartl, W. Majerotto, W. Porod and D. Wyler, Effect of supersymmetric phases on lepton dipole moments and rare lepton decays, Phys. Rev. D 68 (2003) 053005 hep-ph/0306050.
[5] A.G. Cohen, D.B. Kaplan and A.E. Nelson, The more minimal supersymmetric standard model, Phys. Lett. B 388 (1996) 588 hep-ph/9607394;
A.G. Akeroyd, Y.Y. Keum and S. Recksiegel, Effect of supersymmetric phases on the direct CP asymmetry of $B \rightarrow X_{d} \gamma$, Phys. Lett. B 507 (2001) 252 hep-ph/0103008].
[6] S. Yaser Ayazi and Y. Farzan, Reconciling large CP-violating phases with bounds on the electric dipole moments in the MSSM, Phys. Rev. D 74 (2006) 055008 hep-ph/0605272.
[7] D. Chang, W.-Y. Keung and A. Pilaftsis, New two-loop contribution to electric dipole moment in supersymmetric theories, Phys. Rev. Lett. 82 (1999) 900 Erratum Phys. Rev. Lett. 83 (1999) 3972 hep-ph/9811202;
A. Pilaftsis, Higgs-boson two-loop contributions to electric dipole moments in the MSSM, Phys. Lett. B 471 (1999) 174 hep-ph/9909485;
D. Chang, W.-F. Chang and W.-Y. Keung, Additional two-loop contributions to electric dipole moments in supersymmetric theories, Phys. Lett. B 478 (2000) 239 hep-ph/9910465;
A. Pilaftsis, Higgs-mediated electric dipole moments in the MSSM: an application to baryogenesis and Higgs searches, Nucl. Phys. B 644 (2002) 263 hep-ph/0207277.
[8] A. Bartl, K. Hidaka, T. Kernreiter and W. Porod, Impact of CP phases on the search for sleptons stau and sneutrino/tau, Phys. Lett. B 538 (2002) 137 hep-ph/0204071; Tau-sleptons and tau-sneutrino in the MSSM with complex parameters, Phys. Rev. D 66 (2002) 115009 hep-ph/0207186.
[9] A. Bartl, S. Hesselbach, K. Hidaka, T. Kernreiter and W. Porod, Impact of CP phases on stop and sbottom searches, Phys. Lett. B 573 (2003) 153 hep-ph/0307317; Top squarks and bottom squarks in the MSSM with complex parameters, Phys. Rev. D 70 (2004) 035003 hep-ph/0311338.
[10] W. Bernreuther and P. Overmann, Probing Higgs boson and supersymmetry-induced CP-violation in top quark production by (un-)polarized electron positron collisions, Z. Physik C 72 (1996) 461 hep-ph/9511256;
B. Grzadkowski, CP-violation in $t$ anti-t production at $e^{+} e^{-}$colliders, Phys. Lett. B 305 (1993) 384 hep-ph/9303204;
E. Christova and M. Fabbrichesi, The rate difference between the weak decays of $t$ anti-t in supersymmetry, Phys. Lett. B 320 (1994) 299 hep-ph/9307298;
B. Grzadkowski and W.-Y. Keung, SUSY induced CP-violation in $t$ decays at e-e+ colliders, Phys. Lett. B 316 (1993) 137 hep-ph/9306322; The decay rate asymmetry of the top quark, Phys. Lett. B 319 (1993) 526 hep-ph/9310286;
A. Bartl, E. Christova and W. Majerotto, CP-violating asymmetries in top quark production and decay in $e^{+} e^{-}$annihilation within the MSSM, Nucl. Phys. B 460 (1996) 235 hep-ph/9507445;
A. Bartl, E. Christova, T. Gajdosik and W. Majerotto, CP-violating angular asymmetries of $b$ and anti-b quarks in $e^{+} e^{-} \rightarrow t$ anti-t, Phys. Rev. D 58 (1998) 074007 hep-ph/9802352; S. Bar-Shalom, D. Atwood and A. Soni, CP-violation in single top quark production and decay via $p \bar{p} \rightarrow t$ anti- $b+x \rightarrow w+b$ anti- $b+x$ within the MSSM: a possible application for measuring $\arg (a(t))$ at hadron colliders, Phys. Rev. D 57 (1998) 1495 hep-ph/9708357;
D. Atwood, S. Bar-Shalom, G. Eilam and A. Soni, CP-violation in top physics, Phys. Rept. 347 (2001) 1 hep-ph/0006032;
A. Bartl et al., General flavor blind MSSM and CP-violation, Phys. Rev. D 64 (2001) 076009 hep-ph/0103324;
E. Christova, S. Fichtinger, S. Kraml and W. Majerotto, CP-violating asymmetries in single top quark production at the Tevatron p $\bar{p}$ collider, Phys. Rev. D 65 (2002) 094002 hep-ph/0108076;
F. Browning, D. Chang and W.-Y. Keung, CP asymmetry in the Higgs decay into the top pair due to the stop mixing, Phys. Rev. D 64 (2001) 015010 hep-ph/0012258;
E. Christova, H. Eberl, W. Majerotto and S. Kraml, CP-violation in charged Higgs decays in the MSSM with complex parameters, Nucl. Phys. B 639 (2002) 263 [hep-ph/0205227;
CP-violation in charged Higgs boson decays into tau and neutrino, JHEP 12 (2002) 021 hep-ph/0211063;
E. Christova, E. Ginina and M. Stoilov, Supersymmetry through CP-violation in $h+-\rightarrow w+-$ h0, JHEP 11 (2003) 027 hep-ph/0307319;
J.R. Ellis, J.S. Lee and A. Pilaftsis, LHC signatures of resonant $C P$-violation in a minimal supersymmetric Higgs sector, Phys. Rev. D 70 (2004) 075010 hep-ph/0404167;
H. Eberl, T. Gajdosik, W. Majerotto and B. Schrausser, CP-violating asymmetry in chargino decay into neutralino and $w$ boson, Phys. Lett. B 618 (2005) 171 hep-ph/0502112.
[11] G. Valencia, Constructing CP odd observables, hep-ph/9411441.
[12] S.Y. Choi, H.S. Song and W.Y. Song, CP phases in correlated production and decay of neutralinos in the minimal supersymmetric standard model, Phys. Rev. D 61 (2000) 075004 hep-ph/9907474;
A. Bartl, T. Kernreiter and W. Porod, A CP sensitive asymmetry in the three-body decay stau(1) - ं b sneutrino/tau tau+, Phys. Lett. B 538 (2002) 59 hep-ph/0202198];
A. Bartl, H. Fraas, T. Kernreiter and O. Kittel, T-odd correlations in the decay of scalar fermions, Eur. Phys. J. C 33 (2004) 433 hep-ph/0306304;
A. Bartl, T. Kernreiter and O. Kittel, A CP asymmetry in $e^{+} e^{-} \rightarrow$ neutralino(i) neutralino $(j) \rightarrow$ neutralino $(j)$ tau stau $(k)$ with tau polarization, Phys. Lett. B 578 (2004) 341 hep-ph/0309340;
A. Bartl, H. Fraas, O. Kittel and W. Majerotto, CP asymmetries in neutralino production in $e^{+} e^{-}$collisions, Phys. Rev. D 69 (2004) 035007 hep-ph/0308141]; CP sensitive observables in $e^{+} e^{-} \rightarrow$ neutralino(i) neutralino(j) and neutralino decay into z boson, Eur. Phys. J. C 36 (2004) 233 hep-ph/0402016; CP-violation in chargino production and decay into sneutrino, Phys. Lett. B 598 (2004) 76 hep-ph/0406309; CP sensitive observables in chargino production and decay into $a$ W boson, Phys. Rev. D 70 (2004) 115005 (hep-ph/0410054;
S.Y. Choi, M. Drees, B. Gaissmaier and J. Song, Analysis of CP-violation in neutralino decays to tau sleptons, Phys. Rev. D 69 (2004) 035008 hep-ph/0310284;
S.Y. Choi, M. Drees and B. Gaissmaier, Systematic study of the impact of CP-violating phases of the MSSM on leptonic high-energy observables, Phys. Rev. D 70 (2004) 014010 hep-ph/0403054;
J.A. Aguilar-Saavedra, CP-violation in selectron cascade decays selectron(l) $\rightarrow e$ neutralino(2) $\rightarrow e$ neutralino(1) $\mu^{+} \mu^{-}$, Phys. Lett. B 596 (2004) 247 hep-ph/0403243; CP-violation in neutralino(1) neutralino(2) production at a linear collider, Nucl. Phys. B 697 (2004) 207 hep-ph/0404104;
A. Bartl, H. Fraas, S. Hesselbach, K. Hohenwarter-Sodek and G.A. Moortgat-Pick, A t-odd asymmetry in neutralino production and decay, JHEP 08 (2004) 038 hep-ph/0406190;
A. Bartl et al., $C P$-odd observables in neutralino production with transverse e+ and e-beam polarization, JHEP 01 (2006) 170 hep-ph/0510029;
K. Kiers, A. Szynkman and D. London, CP-violation in supersymmetric theories: stop(2) $\rightarrow$ stop (1) tau- tau+, Phys. Rev. D 74 (2006) 035004 hep-ph/0605123;
A. Bartl et al., CP asymmetries in chargino production and decay: the three- body decay case, hep-ph/0608065.
[13] A. Bartl, E. Christova, K. Hohenwarter-Sodek and T. Kernreiter, Triple product correlations in top squark decays, Phys. Rev. D 70 (2004) 095007 hep-ph/0409060.
[14] S. Kawasaki, T. Shirafuji and S.Y. Tsai, Productions and decays of short-lived particles in $e^{+} e^{-}$colliding beam experiments, Prog. Theor. Phys. 49 (1973) 1656.
[15] C.J.S. Damerell, private communication.
[16] C.J. Damerell and D.J. Jackson, Vertex detector technology and jet flavor identification at the future $e+e$ - linear collider, SPIRES entry Prepared for 1996 DPF / DPB Summer Study on New Directions for High-Energy Physics (Snowmass 96), Snowmass, Colorado, 25 Jun - 12 Jul 1996.
SLD collaboration, K. Abe et al., Improved direct measurement of $A_{b}$ and $A_{c}$ at the $Z^{0}$ pole using a lepton tag, Phys. Rev. Lett. 88 (2002) 151801 hep-ex/0111035;
DELPHI collaboration, J. Abdallah et al., b-tagging in delphi at LEP, Eur. Phys. J. C 32 (2004) 185 hep-ex/0311003.
[17] G. Alexander et al., TESLA Technical Design Report, Part IV, 'A Detector for Tesla,' http://tesla.desy.de/new_pages/TDR_CD/PartIV/detect.htm.
[18] J.P. Derendinger and C.A. Savoy, Quantum effects and $\mathrm{SU}(2) \times \mathrm{U}(1)$ breaking in supergravity gauge theories, Nucl. Phys. B 237 (1984) 307;
J.A. Casas and S. Dimopoulos, Stability bounds on flavor-violating trilinear soft terms in the MSSM, Phys. Lett. B 387 (1996) 107 hep-ph/9606237.
[19] Particle Data Group collaboration, S. Eidelman et al., Review of particle physics, Phys. Lett. B 592 (2004) 1.


[^0]:    ${ }^{1}$ In principle, the leptons from the decays (1.3)-(1.5) can be distinguished through their different angular or energy distributions.

